ROLLOVER AND TRANSSHIPMENT PROBLEMS IN OPERATIONS MANAGEMENT

by

Chao Liang

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Dedicated to my parents, family, and teachers.

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by

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DISSERTATION

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PREFACE

This dissertation was produced in accordance with guidelines which permit the inclusion as part of the dissertation the text of an original paper or papers submitted for publication. The dissertation must still conform to all other requirements explained in the "Guide for the Preparation of Master's Theses and Doctoral Dissertations at The University of Texas at Dallas". It must include a comprehensive abstract, a full introduction and literature review and a final overall conclusion. Additional material (procedural and design data as well as descriptions of equipment) must be provided in sufficient detail to allow a clear and precise judgment to be made of the importance and originality of the research reported. It is acceptable for this dissertation to include as chapters authentic copies of papers already published, provided these meet type size, margin and legibility requirements. In such cases, connecting texts which provide logical bridges between different manuscripts are mandatory. Where the student is not the sole author of a manuscript, the student is required to make an explicit statement in the introductory material to that manuscript describing the student's contribution to the work and acknowledging the contribution of the other author(s). The signatures of the Supervising Committee which precede all other material in the dissertation attest to the accuracy of this statement.

ROLLOVER AND TRANSSHIPMENT PROBLEMS

IN OPERATIONS MANAGEMENT

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This dissertation addresses three important problems in Operations Management.

The first problem compares the two different product rollover strategies (single rollover vs. dual rollover) when selling to strategic customers. With single rollover, when a new product is introduced, the old product is phased out from the market. With dual rollover, the old product remains in the market together with the new product. Strategic customers refer to those customers who are forward-looking and consider future purchase opportunities when making the decision. We investigate how customers behave under different product rollover strategies. We also examine how the new product innovation and the proportion of strategic customers in the market impact a firm's rollover strategy decision.

The second problem analyzes the impact of strategic customer behavior and rollover strategies on a firm's product innovation decision. We analytically compare the firm's optimal innovation level and profit in four settings: when the customers are myopic or strategic and when single rollover or dual rollover is adopted for product transitions over time. In contrast with the common wisdom and the extant marketing literature, we find that the strategic waiting

behavior speeds up the innovation process.

The third problem compares site-to-store and store-to-site strategies for dual-channel integration. With site-to-store (resp., store-to-site) strategy, a firm can fill unmet orders in the physical channel (resp., online channel) with the inventory in the online channel (resp., physical channel). We investigate how the product contribution margin, the channel demand distribution shape and the number of retail stores in the physical channel impact the firm's optimal integration strategy. We propose a heuristic to identify the appropriate integration strategy when there are multiple retail stores. We apply our results to a circular spatial model for dual-channel retailing systems.

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CHAPTER 1

INTRODUCTION

I have three essays in my dissertation: Analysis of Product Rollover Strategies in the Presence of Strategic Customers (Chapter 2), Impact of Strategic Customer Behavior and Rollover Strategies on Product Innovation (Chapter 3), and Site-to-Store or Store-to-Site? Application of One-Way Transshipment in Dual-Channel Retailing (Chapter 4). In this chapter, I will briefly introduce them one by one as following.

1.1 Analysis of Product Rollover Strategies in the Presence of Strategic Customers

Frequent product introductions are common especially in consumer electronic and fashion industries. They emphasize the importance of dealing with the leftover inventory of the old product when rolling over to the new product. In Chapter 2, we examine two primary rollover strategies: *single rollover* and *dual rollover*. With dual rollover, a firm keeps the leftover old product in the market and sells it together with the new product. With single rollover, a firm removes the leftover old product and disposes it of outside the market.

We study a monopolistic firm that decides the appropriate rollover strategy between single and dual rollovers. The firm sequentially sells two products (the old product first and then the new product) in a market with both strategic and myopic customers. Myopic customers do not consider future options. They buy the product on the spot as long as their utility surplus of buying at that moment is non-negative. Different from myopic customers, strategic customers are forward-looking. When strategic customers make decisions, they consider not only whether to buy the product, but also when to buy it and which version to buy (i.e., buy the current product or wait for the new product). Therefore, compared to myopic customers, strategic customers have a tendency to wait, either for the new product or for a possible markdown of the leftover old product.

Compared to dual rollover, the primary disadvantage of single rollover is the revenue loss from the leftovers. This is because the disposal value of the leftover items outside the market is usually lower than their selling price in the market. There are two primary drawbacks of dual rollover: *cannibalization effect* and *postponement effect*. With both products in the market, the sale of the new product may be cannibalized by the old product. Also, compared to single rollover, the presence of the old product in the market gives strategic customers more incentive to wait.

We investigate how customers behave under different rollover strategies and how different customer behaviors in turn impact the firm's rollover strategy. We find that the innovation of the new product and the proportion of strategic customer are the key factors determining the firm's optimal rollover strategy. Specifically, the firm should consider adopting single rollover when the new product innovation is low and the proportion of strategic customers is high. Interestingly and counterintuitively, we find that the revenue loss of the leftovers – a so-called primary "disadvantage" of single rollover may help a firm to earn a higher profit compared to dual rollover. In addition, a firm may suffer from a high value disposal option for the old product. Liang et al. (2011a) is a working paper based on Chapter 2.

1.2 Impact of Strategic Customer Behavior and Rollover Strategies on Product Innovation

Innovation is one of the most important processes for firms to create new markets, transform industries, and sustain growth. There is a common saying: either you innovate or you die. Therefore, we cannot overemphasize the importance of innovation.

In addition to the importance of innovation for firms' development, another strong motivation for us to study this question is the inconsistency between what we observe in practice and our typical logic think along with the extant literature. A typical logic think is that since strategic customers are forward-looking, a firm has a lower pricing power with strategic customers than with myopic customers. The lower pricing power reduces the firm's return on innovation investment and then reduces its incentive to innovate. Dhebar (1994) supports this conventional wisdom by showing that strategic customer behavior imposes a demand-side constraint on the rate of product improvement. However, from what we observe in practice, with increasing number of strategic customers in the market, companies in the fashion and high-tech industries are spending a lot in R&D.

In Chapter 3, with this consistency in mind, we take the innovation level as the firm's decision variable and compare the firm's optimal innovation level and profit under four different settings: when customers are myopic or strategic and single rollover or dual rollover is adopted. We show that when customers are myopic, (1) single rollover hurts the firm's profit but accelerates the innovation process and (2) the innovation level and profit cannot be increased simultaneously with any rollover strategy. In contrast, when customers are strategic, the firm can provide a more innovative product while earning a higher profit by adopting an appropriate rollover strategy. This underscores the importance of choosing the appropriate rollover strategy when selling to strategic customers. Surprisingly, we find that the strategic waiting behavior speeds up the innovation process. Liang et al. (2011b) is a working paper based on Chapter 3.

1.3 Site-to-Store or Store-to-Site? Application of One-Way Transshipment in Dual-Channel Retailing

In a few short years, with dramatic development of online shopping, almost all traditional "brick-and-mortar" retailers such as Walmart, J.C. Penney and Target have already opened new virtual online channels and have become "brick-and-click" retailers. To these retailers, multi-channel retailing becomes a business imperative, and how to integrate the online channel with their traditional physical channel is key to the success of their multi-channel retailing strategy.

In Chapter 4, we study two typical dual-channel integration strategies: *site-to-store* and *store-to-site*. With the store-to-site strategy, a retailer use its inventory in the (physical) retail stores to fulfill the online orders when the online warehouse (i.e., the warehouse designed to fulfill the online orders) is out of stock. With site-to-store strategy, a retailer makes the inventory in the online warehouse available to its retail store customers. Specially, a customer who is not able to find the desired product in a retail store, has the option to order the product from the retailer's web site. The customer can pick up the product from the retail store or have it home delivered.

We find that with one retail store, when only one channel should have inventory, it is the channel with stochastically larger or less uncertain demand. Otherwise, with both channels carrying inventory, the optimal channel integration depends on the product contribution margin and the channel demand distribution shape. When there are multiple retail stores, the site-to-store (resp., store-to-site) strategy becomes more attractive for high-margin (resp., low-margin) products with larger number of retail stores. We also find that the integration strategy decision makes a significant difference if at least one of the following conditions is satisfied: (1) the product contribution margin is either very high or very low, (2) the transshipment cost is high, (3) the demand correlation between the two channels is not strongly positive, and (4) the demand uncertainty difference between the two channels is large.

We propose a heuristic that only requires a comparison of the online demand standard deviation and the sum of demand standard deviations of retail stores in identifying an effective integration strategy, when there are multiple retail stores in the physical channel. Finally, we apply our results to a circular spatial model for dual-channel retailing systems and obtain insights on the impact of customer purchasing behavior on integration strategy selection. Liang et al. (2011) is a working paper based on Chapter 4.

CHAPTER 2

ANALYSIS OF PRODUCT ROLLOVER STRATEGIES IN THE PRESENCE OF STRATEGIC CUSTOMERS

2.1 Synopsis

Frequent product introductions are common in many industries, such as consumer electronics, computers and apparel. Fashion retailers usually replenish their stock with new designs at least once per season. Many firms view frequent product introductions as important ways to increase market share and sustain growth. They manage the phase-out of an old product along with the introduction of a new product that replaces the old one.

Ideally, the introduction of the new product should coincide with the depletion of the old product inventory. However, this ideal is difficult to achieve in an uncertain environment. In reality, a firm usually needs to deal with the leftover inventory of the old product when rolling over to the new product. In this chapter, we study two primary rollover strategies: single (-product) rollover and dual (-product) rollover. With dual rollover, the old product remains in the market together with the new product. With single rollover, when the new product is introduced, the old product is phased out from the market and can be disposed of in various ways. Fire sales, dismantling products for spare parts, recycling the material for future use and write-offs are all common ways to dispose of leftover inventory. Also, more and more U.S. companies are relying on overseas markets to clear their leftovers, which can avoid cannibalizing their regular retail channel in the U.S. market (Kavilanz 2008). Using different introduction dates for the new product in different regions worldwide facilitates the implementation of single rollover. Even the equipment that is obsolete in developed countries can be sold in less-developed countries. The leftover inventory can also be sold at discount stores such as TJ Maxx and Marshalls, outlet malls, and Web sites like Overstock.com. Sometimes, firms donate unsold items for charitable tax deductions. For additional approaches to disposing of unsold inventory, see Tibben-Lembke (2004). The main drawback of single rollover is that the revenue from disposing of leftover items under single rollover is usually lower than that under dual rollover.

Dual rollover, despite obtaining a higher price than the disposal value for the leftover old products by keeping them in the market, has two important drawbacks. The first is the *cannibalization effect*. With both products in the market, the old product may cannibalize sales of the new product, especially when the *innovation* (improvement of the new product over the old product) is not very high. The second is the *postponement effect*. Strategic customers are common in markets for durable goods with rapid innovation, like high-tech and fashion products. They time their purchase decisions and select the product that gives them the highest surplus. Instead of purchasing the current version, strategic customers anticipating the new (product) version may wait. Besides, they may also wait for the markdown of the old version. Therefore, when a strategic customer is making the purchase decision, she takes into account the future opportunity of buying the new version and the marked-down old version that is available under dual rollover. Thus, compared to single rollover, the presence of the old product in the market gives strategic customers more incentive to delay their purchase. Articles in the media such as Arends (2010) discuss the rational for waiting and thus increase the number of strategic customers.

In this chapter, we explore the performance of single rollover in eliminating cannibalization and mitigating postponement effects, and answer the following questions: Can a firm use single rollover to earn a higher profit via reducing the two effects? If yes, when can single rollover increase the firm's profit and by how much? We develop a two-period model of a monopolistic firm that sells an old version in period 1 and introduces a new version in period 2 to a market with uncertain size and strategic customers. The firm needs to decide rollover strategy before period 1 as well as prices and ordering quantities in both periods.

Our work contributes to both the strategic customer behavior research in operations management (OM) and the product rollover strategy literature by studying the interplay between strategic waiting behavior and different rollover strategies. The strategic waiting research in operations management most relevant to ours has been conducted by Su and Zhang (2008), Cachon and Swinney (2009), and Lai et al. (2010). They study the mechanisms to mitigate strategic waiting behavior when demand is uncertain. Su and Zhang (2008) consider quantity and price commitments and show that these can be achieved by various contracts in a supply chain. Cachon and Swinney (2009) explore quick response strategy to better match supply and demand to reduce markdown chance. Lai et al. (2010) show that posterior price matching can substantially improve the firm's profit when the fraction of strategic customers is not too low and their valuation decrease over time is neither too low nor too high. In our model, we study how a firm can use single rollover to mitigate strategic customers' waiting behavior for the first time in the OM literature. These three papers and others (Su and Zhang (2009), Su (2008), Cachon and Swinney (2011), Liu and van Ryzin (2008), Aviv and Pazgal (2008), Yin et al. (2009), Prasad et al. (2010), Huang and Van Mieghem (2010), and Özer and Zheng (2011)) usually focus on the scenario where a firm sells a product in the regular season and then marks down the product in the final sale. But unlike our model, they do not discuss if the firm is willing to sell the leftover product in the market with or without markdown, or the product version introductions leading to cannibalization of sales. Toktay et al. (2011), Erzurumlu et al. (2010) and Khawam and Spinler (2011) consider product introduction strategies, but their focus is not product rollover strategy.

Despite their importance, product rollover strategies have only recently received some attention in the literature. Billington et al. (1998) and Erhun et al. (2007) provide managerial insights derived from hands-on experience. To our knowledge, although different terminologies may be used, five papers consider analytical comparisons of different rollover strategies: Levinthal and Purohit (1989), Lim and Tang (2006), Ferguson and Koenigsberg (2007), Arslan et al. (2009), and Koca et al. (2010). The last four focus on the cannibalization effect and/or the product introduction and phase-out times. Furthermore, customers are neither strategic nor explicitly modeled, and so these paper do not consider the important interaction between rollover strategy and strategic waiting behavior. Levinthal and Purohit (1989) consider both cannibalization and postponement effects but with a deterministic demand and without explicitly modeled customers. In a quite different setting, they show that the single rollover is always better than dual rollover. Our model, on the other side, shows that due to demand-supply mismatch, dual rollover can outperform single rollover.

We consider the performance of single rollover under three innovation cases: high, medium and low. We show that in all innovation cases, the firm can increase its profit by adopting single rather than dual rollover under certain conditions, especially when the proportion of strategic customers is high. We find that the innovation level strongly affects single rollover's performance in mitigating waiting behavior. In the literature for only one product version, the firm can often induce all strategic customers to buy early while extracting *all* their utility by using some mechanisms (e.g., posterior price matching in Lai et al. (2010) and price commitment in Su and Zhang (2008)). *Without any old products left in the market under single rollover, we would expect that the firm can achieve the same result* (i.e., induce early purchase while capturing all the utility) as in the literature. However, in the two-version context, this is true only when the replacement of the old version with the new version is not possible. When the innovation is high enough that customers who have already bought the old version are willing to replace it with the new one, strategic customers may still wait even when they certainly know that under single rollover they cannot access the leftover old version. This is because if the market is saturated (i.e., many customers in the market have already bought the old version), instead of keeping the price high to capture all the utility from the customers who have not bought the old version, the firm may find it more profitable to lower the price of the new version to induce replacements. This creates the waiting incentive of strategic customers. That is, with a high innovation, the firm cannot eliminate waiting behavior even though it commits not to sell any leftover old version in the market. In practice, especially in the consumer electronics industry with a saturated market, the price of the new version may sometimes be even less than or equal to the original price (before its markdown) of the old version. This can lead to a higher customer surplus from buying the new version rather than the old version.

Another interesting finding is that the disposal value of the leftover old version under single rollover plays different roles under different innovation levels. With low and medium innovations, the firm benefits from a high disposal value, which is intuitive. However, when the innovation is high, the firm may *suffer from a high disposal value*. While the result appears to be counterintuitive, it arises because a higher disposal value leads directly to higher inventory of the old version, which implies a higher sale in period 1. With more customers who carry over the old version from period 1 to period 2, the market in period 2 is more saturated. So, instead of pricing the new version high to sell it to only the high-end customers who have not bought the old version in period 1, the firm tends to price it low in order to induce replacements. This increases customers' possibility of getting a positive surplus by waiting. Thus, in addition to the direct, economical benefit of disposal value, there is an indirect, behavioral impact as well: higher disposal value aggravates strategic behavior. When the proportion of strategic customers is high, the indirect behavioral impact may outweigh the direct impact and thus results in a lower profit.

We find that depending on the innovation level, the firm has different product introduction policies. Specifically, when the innovation is high, the firm can introduce both versions hoping that customers will purchase both of them. However, when replacements are not possible (i.e., the innovation is low or medium), as long as the innovation of the new version can compensate the firm's profit discounting and the customers' value depreciation over time, the firm should skip the old version to eliminate the cannibalization of the more profitable new version. Roughly speaking, a fast innovating firm can introduce both versions since it can expect to receive payments twice from repeat customers; a moderately innovating firm may consider skipping the old version and introducing the new version directly even though it may need to wait some time for the new version to be ready; as long as customers value the current version, a slowly innovating firm should introduce it as soon as possible to avoid the loss from time depreciation.

2.2 Model Description

We model a profit-maximizing firm that may introduce a product V1 in period 1 and its upgraded version V2 in period 2. Prior to period 1, the firm must choose between two product rollover strategies: single or dual. In either strategy, only V1 is sold in period 1. In period 2, under single rollover, only V2 is available in the market, whereas, under dual rollover, both V2 and the leftover V1 are available.

We set the innovation levels of V1 and V2 as 1 and $(1 + \theta)$, respectively, where $\theta \geq 0$ denotes the additional innovation from V1 to V2. For example, θ may represent the number of new functions introduced in V2. The higher θ is, the higher the customers are willing to pay. We assume that the firm makes a credible commitment of its rollover strategy prior to period 1. This is reasonable in view of the fact that the rollover strategy can be verified ex post and the firm is averse to the loss of reputation resulting from reneging. Also, a rollover strategy requires arrangements in advance. For example, the firm may need to plan for the required shelf space for the potential leftover V1 in period 2. In single rollover, if the phased out V1 will be sold in overseas markets, then resources such as transportation and storage space may have to be lined up ahead of time. Thus, any deviation from a committed strategy at the last moment could be prohibitively expensive.

The market consists of three distinct customer segments: *strategic customers, myopic customers,* and *bargain hunters*. Customers are assumed to be homogenous within each segment. Strategic and myopic customers are *high-end customers*, and bargain hunters are *low-end customers*. High-end customers' valuation of using V1 in both periods is v and their valuation of using V1 in period 2 is βv, where $0 < \beta < 1$. The parameter β captures the value depreciation due to negative feeling of not being among the first adopters of the product and/or the loss of utility in period 1. Consequently, high-end customers' valuation of V2 in period 2 is $\beta v(1+\theta)$. The market size for high-end customers is a random variable N with c.d.f. $F(\cdot)$ and density $f(\cdot)$. We assume that a fraction ϕ of high-end customers are strategic and the rest are myopic.

Each strategic customer holds a belief of the *waiting surplus* W_c – the surplus that every strategic customer can get by waiting. When making buy/wait decisions in period 1, each strategic customer takes into account the option of buying in period 2. Throughout the chapter, we use the terms *buying now* and *waiting* for strategic customers' buying and not buying V1 in period 1, respectively. Myopic customers, unlike strategic customers, decide to buy or not buy in period 1 without considering the waiting option. We assume that customers buy in the current period whenever the surpluses of buying now and waiting are tied or the surpluses of buying and not buying are tied. Therefore, strategic customers buy V1 in period 1 if the surplus $(v - p_1)$ of buying now, where p_1 is the price of V1 in period 1, is *not less than* the waiting surplus W_c. We should point out that when buying now, strategic customers have the option of replacing V1 with V2 in period 2. However, V2 brings in a lower utility $\beta v\theta$ to the repeat customers compared to the utility $\beta v(1 + \theta)$ to the customers who do not buy in period 1. This implies that replacement, if any, leads to non-positive surplus. This is consistent with the pricing strategies in period 2 as shown in Propositions 2.3.1, 2.3.3, 2.4.1 and 2.4.3. Therefore, we keep the surplus of buying now as $(v - p_1)$, instead of introducing another belief for the replacement surplus, which would be equal to zero in equilibrium. Myopic customers buy V1 in period 1 if their surplus $(v - p_1)$ is *non-negative*. Consequently, $(v - W_c)$ and v are strategic customers' and myopic customers' reservation prices in period 1, respectively.

Bargain hunters, unbounded in number, value the product so low that they only buy it on sale. Therefore, nothing is sold to these customers under single rollover. On the other hand, under dual rollover, they buy V1 in period 2 if the marked-down price is *not larger than* their value δ for the leftover V1. We assume $0 \leq \delta < \beta v$, where we recall that βv is the high-end customers' value for V1 in period 2.

In our context, high-end customers can be grouped differently in each period. In period 1, the groups are myopic customers and strategic customers as mentioned above. In period 2, without a next period to plan for, all customers are myopic. Instead, the purchase history of the customers can be used to split them into two groups. Some customers do not buy V1 in period 1 and the others do. The former and latter customers are termed, respectively, as *period-1 non-buyers* (P1NB) and *period-1 buyers* (P1B).

We assume that the unit production cost c is the same for both versions. This is reasonable particularly for high-tech and fashion products. For example, although the price of a 16GB iPad with both Wi-Fi and 3G was \$130 higher than that of a 16GB iPad with Wi-Fi only, the production cost \$306.50 of the former was just \$16 higher than \$290.50 of the latter (Keizer 2010). Under single rollover, in period 2 the firm receives the disposal value $\sigma \geq 0$ from each leftover V1. Under dual rollover, in period 2 the lowest price that the firm will charge for any leftover V1 is δ, since bargain hunters clear all the leftover V1 at such a price. We assume $\sigma, \delta < c$ to avoid ordering an infinite amount. We also assume $\sigma < \delta$ as the disposal value of the leftover V1 under single rollover is usually lower than its selling price in the market. We refer to $(\delta - \sigma)$ as the *market-disposal spread*. Inventory remaining at the end of period 2 has zero value. The firm discounts the profit in period 2 by a factor of α, $0 < \alpha < 1$.

The firm knows v, β, θ, ϕ, σ, δ, c, and $F(.)$, high-end customers know v, β, and θ, and bargain hunters know δ. The firm holds a belief R_f of the *strategic customers' reservation price* for V1 in period 1. Privately formed, R_f is not accessible to customers, nor is W_c to the firm.

2.2.1 Sequence of Events

The sequence of events is specified in Figure 2.1. Before period 1, the firm announces its rollover strategy. Holding a belief R_f in period 1, the firm decides price p_1 and stocking level q_1 for V1. Strategic customers consider the future options, so $R_f \leq v$. In addition, customers do not buy if the price is higher than their reservation price, so we must have $p_1 \leq v$. For $R_f < v$, the period-1 demand is

$$D_1 = \begin{cases} N & \text{if } p_1 \leq R_f, \\ (1-\phi)N & \text{if } R_f < p_1 \leq v. \end{cases} \tag{2.1}$$

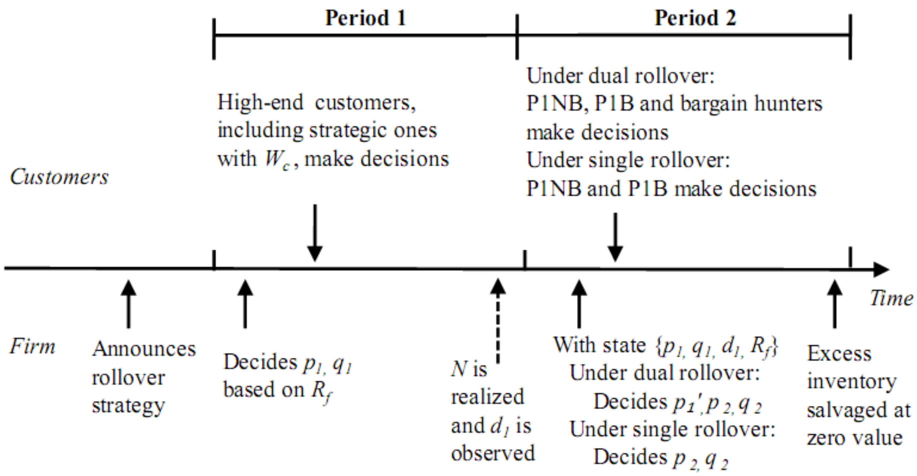


Figure 2.1. Sequence of Events

The sales in period 1 is $S = \min\{q_1, D_1\}$. Hence, the firm sets either $p_1 = v$ or $p_1 = R_f$ to maximize its total profit. If $R_f = v$, then $D_1 = N$. Because $R_f = v$ occurs only in special cases, we focus on $R_f < v$ and deal with the special cases as they occur.

After the firm announces its rollover strategy and p_1, all high-end customers arrive in period 1. They observe p_1, but not q_1. Should they find V1 in stock, strategic customers decide whether to buy now or wait and myopic customers decide whether to buy now or not, based on their respective reservation prices.

N and correspondingly S and D_1 are realized at the end of period 1, respectively, as n, s and d_1. We assume that the firm can assess the demand d_1 of V1 at the end of period 1. When there is no stockout, d_1 equals the sales s. When some customers cannot get V1 due to stockout, their visits, inquiries and complaints help the firm to estimate d_1. Since the firm sets p_1 and observes $d_1 \in \{(1-\phi)n, n\}$, it can deduce the realized n from d_1 at the end of period 1 by

$$n = \begin{cases} d_1 & \text{if } p_1 = R_f, \\ d_1/(1-\phi) & \text{if } p_1 = v. \end{cases} \tag{2.2}$$

In period 2, if single rollover is adopted, then the firm decides price p_2 and stocking level

q_2 for V2; if dual rollover is adopted, then it decides p_2, q_2 for V2 and marked-down price p_1' for the leftover V1.

Under single rollover, P1NB buy V2 if the surplus $\beta v(1+\theta)-p_2$ is nonnegative; otherwise, they buy nothing. Under dual rollover, P1NB buy V2, V1 or nothing based on their surplus rankings. For example, if $\beta v(1+\theta)-p_2 > \beta v - p_1' > 0$, then P1NB prefer V2 to V1. Buying nothing, which provides zero surplus, is their last option. When both the leftover V1 and V2 give P1NB the same surpluses, that is $\beta v(1+\theta)-p_2 = \beta v - p_1'$, we assume that the firm can induce them to buy whichever version the firm prefers to sell. One explanation is that a salesperson can influence a customer's purchase decision when she is indifferent between these two versions. In the remainder of the chapter, we say that P1NB prefer a particular version when they have a higher surplus from purchasing that than the other, or when they are indifferent between the two and the firm induces them to buy the particular version. Under both rollover strategies, P1B purchase V2 to replace V1 if the incremental value $\beta v(1+\theta) - \beta v$ from V1 to V2 is greater than or equal to p_2, that is $p_2 \leq \beta v\theta$; otherwise, they continue using V1 in period 2. Bargain hunters can afford V1 if $p_1' \leq \delta$.

When both P1NB and bargain hunters want V1 in period 2, P1NB have purchase priority over bargain hunters. Similarly, P1NB have purchase priority over P1B. Customers within each group have equal priority. Similar prioritization is used in Su and Zhang (2008), Cachon and Swinney (2009), and Lai et al. (2010). In addition, under dual rollover, P1NB who have two versions to choose from can switch to their second best version if they cannot get their preferred version due to a stockout, as we formalize in (2.13) and (2.14) below. Table 2.1 summarizes the customers' decisions.

Table 2.1. Customers' Decision

	Period 1		Period 2	
	Single/Dual Rollovers		Single Rollover	Dual Rollover
Strategic customers	Wait	P1NB	V2 or nothing	V1 or V2 or nothing
	Buy V1	P1B	V2 or keep using V1	V2 or keep using V1
Myopic customers	Not buy	P1NB	V2 or nothing	V1 or V2 or nothing
	Buy V1	P1B	V2 or keep using V1	V2 or keep using V1
BH	N/A	BH	N/A	V1 or nothing

2.2.2 Rational Expectation Equilibrium (REE)

We first describe the equilibrium under single rollover. In period 2, the firm decides q_2 and p_2 by solving

$$\text{Firm's optimality in period 2:} \quad (q_2(y), p_2(y)) \in \arg\max_{q_2,\, p_2} \Pi_2^S(q_2, p_2|y), \tag{2.3}$$

where $\Pi_2^S(q_2, p_2|y)$ is the firm's profit in period 2 and the firm's information set at the end of period 1 is $y = \{q_1, p_1, d_1, R_f\}$. We let $Y = \{q_1, p_1, D_1, R_f\}$. In period 2, all customers are myopic and their alternatives are summarized in Table 1. Because customers' decisions in period 2 are relatively straightforward given the prices, we implicitly incorporate their decisions through the firm's demands in period 2.

Holding a belief R_f in period 1, the firm's expected total profit is $\mathbf{E}[\Pi^S(q_1, p_1)] = \mathbf{E}[p_1 \min\{q_1, D_1\} - cq_1 + \alpha \max_{q_2,p_2} \Pi_2^S(q_2, p_2|Y)]$, where the first two terms together denote the firm's profit in period 1. The firm decides q_1 and p_1 by solving the firm's optimality problem: $(q_1(R_f), p_1(R_f)) \in \arg\max_{q_1,p_1} \mathbf{E}[\Pi^S(q_1, p_1)]$. Because we have only two possible values for p_1 (v or R_f), we can break down the firm's optimization problem in period 1 into the following two optimization problems:

$$\text{Firm's quantity optimality in period 1:} \begin{cases} \text{If } p_1 = v, & q_1(R_f) \in \arg\max_{q_1} \mathbf{E}[\Pi^S(q_1, v)], \\ \text{If } p_1 = R_f, & q_1(R_f) \in \arg\max_{q_1} \mathbf{E}[\Pi^S(q_1, R_f)], \end{cases} \tag{2.4}$$

$$\text{Firm's pricing optimality in period 1: } p_1(R_f) \in \arg\max_{p_1 \in \{v, R_f\}} \mathbf{E}[\Pi^S(q_1(R_f), p_1)]. \tag{2.5}$$

We use $\chi \in \{0, 1\}$ to capture strategic customers' purchase decisions in period 1. $\chi = 1$ and $\chi = 0$, respectively, denote buying now and waiting. We have the following condition

$$\text{Strategic customer's optimality:} \quad \chi = 1 \quad \Longleftrightarrow \quad v - p_1 \geq W_c. \tag{2.6}$$

As myopic customers always buy early with either value of p_1, we have $D_1 = (1-\phi)N + \chi\phi N$.

We adopt the commonly used concept of rational expectation equilibrium. Under rational expectations, beliefs W_c and R_f must be consistent with their outcomes. So, for any given q_1, p_1, χ, W_c and R_f, which satisfy (2.4)-(2.6), the two conditions must hold:

$$\text{Waiting surplus rational expectation: } W_c = w(q_1, p_1, \chi), \tag{2.7}$$

$$\text{Reservation price rational expectation: } R_f = v - W_c, \tag{2.8}$$

where $w(q_1, p_1, \chi)$ is the (actual) expected waiting surplus for each waiting customer. From (2.7), the strategic customers' belief of the waiting surplus is consistent with the expected waiting surplus in equilibrium. According to (2.8), the firm's belief of the strategic customers' reservation price is consistent with the reservation price from the strategic customers' point of view.

Throughout the chapter, if there are period-2 functions $(q_2(y), p_2(y))$ and period-1 numbers $(q_1, p_1, \chi, W_c,\ R_f)$ satisfying conditions (2.3)-(2.8), we call it a *rational expectation equilibrium (REE) under single rollover*. An *REE under dual rollover* consists of period-2 functions $(q_2(y), p_2(y), p_1'(y))$ and period-1 numbers $(q_1, p_1, \chi, W_c, R_f)$ satisfying conditions (2.6)-(2.8) and (2.9)-(2.11):

$$\text{Firm's optimality in period 2:} \quad (q_2(y), p_2(y), p_1'(y)) \in \arg \max_{q_2,\, p_2,\, p_1'} \Pi_2^D(q_2, p_2, p_1'|y). \tag{2.9}$$

$$\text{Firm's quantity optimality in period 1: } \begin{cases} \text{If } p_1 = v, & q_1(R_f) \in \arg\max_{q_1} \ \mathbf{E}[\Pi^D(q_1, v)], \\ \text{If } p_1 = R_f, & q_1(R_f) \in \arg\max_{q_1} \ \mathbf{E}[\Pi^D(q_1, R_f)], \end{cases} \tag{2.10}$$

$$\text{Firm's pricing optimality in period 1: } p_1(R_f) \in \arg \max_{p_1 \in \{v, R_f\}} \ \mathbf{E}[\Pi^D(q_1(R_f), p_1)], \tag{2.11}$$

where $\mathbf{E}[\Pi^D(q_1, p_1)] = \mathbf{E}[p_1 \min\{q_1, D_1\} - cq_1 + \alpha \max_{q_2, p_2, p_1'} \Pi_2^D(q_2, p_2, p_1'|Y)]$.

The (actual) expected waiting surplus $w(q_1, p_1, \chi)$ for each waiting customer can be computed by

$$w(q_1, p_1, \chi) = \int_0^\infty w(q_1, p_1, \chi, q_2, p_2, p_1'|n) f(n) dn, \tag{2.12}$$

where $w(q_1, p_1, \chi, q_2, p_2, p_1'|n)$ is the (actual) average waiting surplus for each waiting customer given the market size n. We use the dependence of q_2, p_2 and p_1' on q_1, p_1, χ and n when performing the integration. We use the word "average" because of the rationing among customers as detailed in (2.13)-(2.15) below. For single rollover, we set $p_1' = \infty$ in view of the fact that there is no V1 available in period 2.

Under dual rollover, if neither version gives P1NB a positive surplus, then obviously we have $w(q_1, p_1, \chi, q_2, p_2, p_1'|n) = 0$. Otherwise, at least one version gives P1NB a positive surplus, and $w(q_1, p_1, \chi, q_2, p_2, p_1'|n)$ is computed by (2.13) or (2.14) below. If P1NB prefer the leftover V1 to V2 in period 2, then

$$\begin{aligned} & w(q_1, p_1, \chi, q_2, p_2, p_1'|n) \\ = \; & \underbrace{\min\{\frac{q_1 - s}{n - s}, 1\}}_{\text{Prob. of getting V1}}(\beta v - p_1') \\ & + \underbrace{[1 - \min\{\frac{q_1 - s}{n - s}, 1\}] \min\{\frac{q_2}{(n - q_1)^+}, 1\}}_{\text{Prob. of not getting V1 but getting V2}}[\beta v(1 + \theta) - p_2], \end{aligned} \tag{2.13}$$

where $(q_1 - s)$ is the number of leftover V1, $(n - s)$ is the number of P1NB, and $(n - q_1)^+ = ((n - s) - (q_1 - s))^+$ is the number of P1NB who cannot get V1 due to a stockout in period 2. If P1NB prefer V2 to the leftover V1 in period 2, then

$$\begin{aligned} & w(q_1, p_1, \chi, q_2, p_2, p_1'|n) \\ = \; & \underbrace{\min\{\frac{q_2}{n - s}, 1\}}_{\text{Prob. of getting V2}}[\beta v(1 + \theta) - p_2] \\ & + \underbrace{[1 - \min\{\frac{q_2}{n - s}, 1\}] \min\{\frac{q_1 - s}{(n - s - q_2)^+}, 1\}}_{\text{Prob. of not getting V2 but getting V1}}(\beta v - p_1'), \end{aligned} \tag{2.14}$$

where $(n - s - q_2)^+$ is the number of P1NB who cannot get V2 due to a stockout.

Under single rollover, because there is no V1 in the market, P1NB can only purchase V2.

Therefore, given the market size n, if the utility surplus of purchasing V2 is non-negative, we have

$$w(q_1, p_1, \chi, q_2, p_2, \infty|n) = \min\{\frac{q_2}{n-s}, 1\}[\beta v(1+\theta) - p_2]. \tag{2.15}$$

We divide the value of innovation θ into three ranges: high $\theta \geq c/\beta v$, medium $(c-\delta)/\beta v \leq \theta < c/\beta v$, and low $\theta < (c-\delta)/(\beta v)$. P1B purchase V2 only if $p_2 \leq \beta v\theta$, where $\beta v\theta$ is the incremental value of replacing V1 with V2. The firm can set $p_2 = \beta v\theta$ profitably only when $\beta v\theta \geq c$, which gives the first critical value $(c/\beta v)$ for θ. Under dual rollover, if V1 is priced low as δ to target bargain hunters in period 2 and V2 is priced high, P1NB may go for V1 to get the surplus $(\beta v - \delta)$ rather than purchase V2. To attract P1NB from V1, the highest price which the firm can charge for V2 is $(\beta v\theta + \delta)$. The firm can set $p_2 = \beta v\theta + \delta$ profitably only when $\beta v\theta + \delta \geq c$, which provides the second critical value $(c-\delta)/\beta v$ for θ.

Before our analysis, we make some observations about possible prices in period 2. In period 2 there are only two possible prices βv and δ for any leftover V1 and only three possible prices $\beta v\theta$, $\beta v\theta + \delta$, and $\beta v(1+\theta)$ for V2 in equilibrium. Intuitively, βv and δ are the highest prices for V1 to target P1NB and bargain hunters in period 2, respectively. Similarly, $\beta v\theta$, $\beta v\theta + \delta$, and $\beta v(1+\theta)$ are the highest prices for V2 in period 2, to target P1B, to attract P1NB from V1 when $p_1' = \delta$, and to target P1NB, respectively. For ease of exposition, we refer to βv and δ as high (H) and low (L) price for V1, respectively, and $\beta v(1+\theta)$, $\beta v\theta+\delta$, and $\beta v\theta$ as high (H), medium (M), and low (L) price for V2, respectively, in period 2.

2.3 Low and Medium Innovations

With low ($\theta < (c-\delta)/(\beta v)$) and medium ($(c-\delta)/\beta v \leq \theta < c/\beta v$) innovations, it is not profitable for the firm to price V2 low enough to target P1B. Consequently, there are two groups of customers ($(n-s)$ P1NB and unlimited bargain hunters) to consider in period 2. There are $(q_1 - s)$ units of leftover V1 from period 1. Period-1 non-buyers choose from buying V1, V2, or nothing, depending on their surplus rankings as we discuss in Section

2.2.1. Bargain hunters cannot afford V2 and buy V1 only if its price is less than or equal to δ.

2.3.1 Dual Rollover

We assume $\theta \geq (c - \beta v)/(\beta v)$, that is, $c \leq \beta v(1+\theta)$ to avoid the case in which V2 cannot be profitably offered. We present the low innovation case below in detail and later discuss only the difference between low and medium innovation cases for brevity. We analyze the problem in a backward manner.

The third row of the table in Proposition 2.3.1 below deals with the firm's pricing and stocking level decisions in period 2 in four cases with low innovation. The four cases are characterized according to the price $p_1 \in \{R_f, v\}$ as well as the relationship between q_1 and the realized market size n at the end of period 1. Also obtained in Proposition 2.3.1 are the expected waiting surpluses (the fourth row of the table) in each of the two cases of p_1, provided that the firm follows the pricing and stocking level decisions listed in the third row of the table.

Proposition 2.3.1 *The firm's period-2 optimal decisions are:*

$p_1 = R_f$		$p_1 = v$	
$n \leq q_1$	$n > q_1$	$n \leq \frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1$	$n > \frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1$
L-H	H-H	L-H	H-H
$w(q_1, R_f, 1) = (\beta v - \delta)F(q_1)$		$w(q_1, v, 0) = (\beta v - \delta)F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right)$	

In Proposition 2.3.1, L-H denotes $p_1' = \delta$, $p_2 = \beta v(1+\theta)$ and $q_2 = (n - q_1)^+$ with the corresponding profit $\Pi_2^D = [\beta v(1+\theta) - c](n - q_1)^+ + \delta(q_1 - s)$; H-H denotes $p_1' = \beta v$, $p_2 = \beta v(1+\theta)$ and $q_2 = (n - q_1)^+$ with $\Pi_2^D = [\beta v(1+\theta) - c](n - q_1)^+ + \beta v[\min\{n, q_1\} - s]$. Note that the nomenclature for prices is consistent with our discussion at the end of Section 3.2, and similar naming conventions are used later.

According to Proposition 2.3.1, with low innovation the firm should always price V2 high and leave a surplus of zero to the waiting customers in period 2. Comparing the conditions for the two pricing strategies, we know that when the high-end market is highly saturated ($n \leq q_1$ when $p_1 = R_f$ and $n \leq \frac{\delta}{\beta v\phi+\delta(1-\phi)} q_1$ when $p_1 = v$), the firm should price V1 low to target bargain hunters in period 2. Consequently, strategic customers get a positive surplus by delaying their purchases. When the high-end market is less saturated, the firm prices V1 high to leave the waiting customers with a surplus of zero. Therefore, when the innovation is low, the positive surplus, if any, is always obtained from the deeply marked-down V1.

To ensure the existence of an REE, we assume that the high-end market size N satisfies the monotone scaled likelihood ratio (MSLR) property, i.e., for any $\kappa \in [0, 1]$ and x in the support of N, $f(\kappa x)/(f(x))$ is monotone in x. The MSLR property is satisfied by gamma, Weibull, uniform, exponential, power, beta, chi, and chi-squared distributions. It has also been assumed in Cachon and Swinney (2009) and Lai et al. (2010).

Period-1 price can be high $p_1 = v$ or low $p_1 = R_f$. These two cases are detailed in Lemmas 6.1.1 and 6.1.2 in the appendix. Proposition 2.3.2 below characterizes the REE under dual rollover and the firm's optimal price in period 1 by comparing its profits under the high and low prices. Here the superscript L, D means "Low innovation and dual rollover". We use superscript * to denote the values in an REE.

Proposition 2.3.2 (Low Innovation) *Under dual rollover, there exists a unique REE. In addition, a $\phi^{L,D}$ exists such that if $\phi \leq \phi^{L,D}$, then the firm sets the high price $p_1^* = v$; otherwise, it sets the low price $p_1^* = R_f$.*

The firm chooses between lowering the price in period 1 to induce all high-end customers to buy and keeping the price high to induce only myopic customers to buy. Proposition 2.3.2 states that the firm should focus on the "majority" of the high-end market. Whether the majority of the market is strategic or myopic customers depends on the relation between ϕ and the critical value $\phi^{L,D}$. If $\phi \leq \phi^{L,D}$, then the majority is myopic and the firm should focus on them by setting a high price; otherwise, the firm should set a low price to induce strategic as well as myopic customers to buy early. The value of $\phi^{L,D}$ is investigated in Section 2.5.

The analysis and insights obtained for medium innovation are similar to those of low innovation. One difference is that with medium innovation, the firm can profitably charge the medium price $(\beta v\theta + \delta)$ for V2 to attract P1NB to buy V2 rather than V1, even when V1 is priced low at δ. Consequently, both V1 and V2 can give waiting customers a positive surplus.

2.3.2 Single Rollover

Unlike dual rollover, leftover V1 is not in the market in period 2 under single rollover. With low and medium innovations, the firm cannot sell V2 to P1B. Consequently, the firm always targets P1NB in period 2. Without the cannibalization from V1, the firm has an absolute pricing power for V2 and sets $p_2 = \beta v(1+\theta)$, leaving a surplus of zero to P1NB. Detailing these, Proposition 2.3.3 is analogous to Proposition 2.3.1.

Proposition 2.3.3 (Low and Medium innovations) *The price for V2, the stocking level for V2, and the firm's profit in period 2 are* $p_2 = \beta v(1+\theta)$, $q_2 = n - s$, *and* $\Pi_2^S = [\beta v(1+\theta) - c](n-s) + \sigma(q_1 - s)$, *respectively.*

With $p_2 = \beta v(1+\theta)$ in period 2, the expected waiting surplus is zero. Then the reservation price for strategic customers in period 1 is v, the same as that for myopic customers, and thus every high-end customer (strategic or myopic) wants to buy under $p_1 = v$. Therefore, we have an important result: single rollover *completely* eliminates the waiting surplus with low and medium innovations.

With low innovation and dual rollover, waiting customers can derive a positive surplus from the deeply marked-down V1. Removing the leftover V1 from the market reduces the waiting surplus to zero.

With medium innovation and dual rollover, both the deeply marked-down V1 and the less aggressively priced V2 can give waiting customers positive surpluses. Single rollover can still completely eliminate the waiting surplus because of its direct and indirect influences. Directly, waiting customers cannot purchase the deeply marked-down V1, as it is absent. Indirectly, without the cannibalization from V1 in period 2, the firm can set the high price $\beta v(1+\theta)$ for V2 to capture the entire surplus of the waiting customers. Thus, *the cannibalization and postponement effects are not independent.* A lower or no cannibalization strengthens the firm's pricing power and thus leads to a higher price for the new version, which subsequently reduces the postponement effect. Proposition 2.3.4 provides the firm's optimal decisions in period 1.

Proposition 2.3.4 (Low and Medium innovations) *Under single rollover, there exists a unique REE, where* $p_1^* = v$*, and* $q_1^* = F^{-1}(\frac{v-c-\alpha(\beta v(1+\theta)-c)}{v-\alpha[\beta v(1+\theta)-c+\sigma]})$ *if* $v > c+\alpha[\beta v(1+\theta)-c]$ *and* $q_1^* = 0$ *otherwise.*

2.3.3 Optimal Rollover Strategy

Proposition 2.3.5 (Low and Medium innovations) *i) If* $c+\alpha[\beta v(1+\theta)-c] \geq v$*, then the firm does not introduce V1, and single rollover and dual rollover give the same profit.*
ii) If $c+\alpha[\beta v(1+\theta)-c] < v$*, then a threshold* $\triangle \geq 0$ *exists such that*
ii.a) for market-disposal spread $\delta-\sigma \leq \triangle$*, a* ϕ^{LM} *exists such that single rollover is optimal iff* $\phi \geq \phi^{LM}$*;*
ii.b) for market-disposal spread $\delta-\sigma > \triangle$*, dual rollover is optimal.*

Proposition 2.3.5 i) shows that if $c+\alpha[\beta v(1+\theta)-c] \geq v$, the firm should offer only V2 and skip V1 (i.e., $q_1 = 0$). Rewriting $c+\alpha[\beta v(1+\theta)-c] \geq v$ as $\alpha[\beta v(1+\theta)-c] \geq v-c$, we see that $\alpha[\beta v(1+\theta)-c]$ is the discounted maximum unit profit by selling V2 only, while $(v-c)$ is the maximum unit profit by selling V1 only. With low and medium innovations, the two versions are so similar that it is not profitable for the firm to sell both versions to the same high-end customers, i.e., replacement is not possible. Selling one more unit of V1 to high-end customers means the number of V2 the firm can sell will decrease by one. Thus, to eliminate the cannibalization, as long as the innovation can compensate the firm's profit discounting and the customers' value depreciation over time ($\alpha[\beta v(1+\theta)-c] \geq v-c$), the firm should be patient to wait until the new technology for V2 is ready and introduce V2 directly. This result applies to a firm that dominates the market, or has leading technologies compared to its competitors. In contrast, in a highly competitive market, the time to market is vital, and rival firms may launch their own products to capture the market while another firm is waiting for the new technology. See Özer and Uncu (2012) for more discussion on time to market. Proposition 2.3.5 ii) shows that if the compensation from innovation is not enough $\alpha[\beta v(1+\theta)-c] < v-c$, the firm should introduce V1 to avoid the loss due to time depreciation.

Proposition 2.3.5 ii) summarizes the conditions under which single rollover can increase the firm's profit: the market-disposal spread ($\delta-\sigma$) cannot be too large, and the proportion of the strategic customers cannot be too low. The disadvantage of single rollover is the lower revenue from the leftover V1 in period 2, while its advantages include guaranteeing the high price $p_2 = \beta v(1+\theta)$ for V2 in period 2 by eliminating the cannibalization from V1, and empowering the firm to set the high price $p_1 = v$ for V1 in period 1 to induce not only myopic customers but also strategic customers to buy early. When the market-disposal spread is low, the loss from the lower revenue of the leftover V1 is small. When the proportion of strategic customers is high, their waiting behavior has a significant impact on the firm's profit. In this case, the gain from eliminating their waiting behavior may more than compensate for the loss from the lower revenue of the leftover V1.

2.4 High Innovation

With high innovation ($\theta \geq c/\beta v$), the incremental value gained from replacing V2 with V1 can justify the production cost c, and thus the firm can profitably price V2 to target P1B. This makes the pricing and stocking level decisions in period 2 more interesting and complicated to analyze. Furthermore, when $p_1 = R_f$, a solution satisfying the REE conditions does not always exist as we show below.

2.4.1 Dual Rollover

Proposition 2.4.1 *The firm's period-2 optimal decisions are:*

$p_1 = R_f$		$p_1 = v$					
every ϕ		$\phi \leq \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$		$\frac{\beta v\theta - c}{\beta v(1+\theta)-c} < \phi \leq \frac{\beta v\theta - c}{\beta v\theta + \delta - c}$		$\phi > \frac{\beta v\theta - c}{\beta v\theta + \delta - c}$	
$n \leq Aq_1$	$n > Aq_1$	$n \leq Aq_1$	$n > Aq_1$	$n \leq Bq_1$	$n > Bq_1$	$n \leq Cq_1$	$n > Cq_1$
L-L	H-H	L-L	H-H	L-L	H-H	L-M	H-H
$w(q_1, R_f, 1) = \beta v F(Aq_1)$		$w(q_1, v, 0) = \beta v F(Aq_1)$		$w(q_1, v, 0) = \beta v F(Bq_1)$		$w(q_1, v, 0) = (\beta v - \delta) F(Cq_1)$	

where $A = \frac{\beta v(1+\theta)-c}{\beta v}$, $B = \frac{\delta}{\beta v - [\beta v(1+\theta)-\delta-c](1-\phi)}$, $C = \frac{\delta}{\delta + (\beta v - 2\delta)\phi}$, *and* L-L *denotes* $p_1' = \delta$, $p_2 = \beta v\theta$ *and* $q_2 = n$ *with* $\Pi_2^D = (\beta v\theta - c)n + (q_1 - s)\delta$*;* L-M *denotes* $p_1' = \delta$, $p_2 = \beta v\theta + \delta$ *and* $q_2 = n - s$ *with* $\Pi_2^D = [\beta v\theta + \delta - c](n-s) + \delta(q_1 - s)$*;* H-H *denotes* $p_1' = \beta v$, $p_2 = \beta v(1+\theta)$ *and* $q_2 = n - s$ *with* $\Pi_2^D = [\beta v(1+\theta) - c](n - s)$*.*

According to Proposition 2.4.1, when $p_1 = v$, the strategies in period 2 depend on the proportion of strategic customers, while with $p_1 = R_f$, the strategies hold for every ϕ. With high innovation, the firm can set the price of V2 as low as $\beta v\theta$ to induce P1B to go for replacements. Similar to medium innovation, the positive surplus can come from either V1 or V2, but V2 is always preferred by P1NB as seen from the proof of Proposition 2.4.1.

With $p_1 = R_f$ in high innovation, the firm's total profit $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ may have one or two maximizers, which is different in low and medium innovations. In addition, a vector of numbers $(q_1, p_1, \chi, W_c, R_f)$ satisfying the REE conditions does not always exist when $\arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ has two values. In such a case, we resort to a mixed strategy in terms of the stocking level q_1. That is, the firm stocks q_1^- with probability λ and q_1^+ with probability $(1-\lambda)$, where q_1^- and q_1^+ are the smaller and larger maximizer of $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ given p_1, respectively. Consequently, the waiting surplus in (2.7) is $w(q_1, p_1, \chi) = \lambda w(q_1^-, p_1, \chi) + (1-\lambda) w(q_1^+, p_1, \chi)$. Except for this, there is no change in the REE conditions. We should note that this mixed strategy is only "partially mixed" since the firm only mixes two stocking levels and chooses the same p_1 for both q_1^- and q_1^+. Proposition 2.4.2 summarizes the result with $p_1 = R_f$.

Proposition 2.4.2 *With $p_1 = R_f$, there is either a unique solution $(q_1, p_1, \chi, W_c, R_f)$ or a combination of $(q_1^-, p_1, \chi, W_c, R_f)$ and $(q_1^+, p_1, \chi, W_c, R_f)$ satisfying the REE conditions except for* (2.11)*. In the latter case, $q_1^- = 0$, and the firm's total profit is $\alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$.*

In Proposition 2.4.2, $q_1^- = 0$, so the firm does not introduce V1 with probability λ. If $p_1 = R_f$ gives the firm a higher profit compared with $p_1 = v$ and the firm mixes two stocking levels, we refer to $p_1 = R_f$ and the associated solution as a *zero-order-mixed REE under dual rollover*. Similarly, we can define a *zero-order-mixed REE under single rollover* which is used in Section 2.4.2.

Surprisingly, we find that under high innovation, the firm can still introduce V1 even when $c + \alpha[\beta v(1+\theta) - c] \geq v$. This is different from low and medium innovation cases, in which the firm skips V1. With low and medium innovations, the two versions are so similar that the firm cannot sells both of them to the same high-end customer. With high innovation, however, the two versions are quite different. By setting $p_1 = v$ and $p_2 = \beta v\theta$, the firm can sell both to the same customer and earn a profit $v - c + \alpha[\beta v\theta - c]$. This profit may be greater than $\alpha[\beta v(1+\theta) - c]$, the maximum unit profit by selling V2 only. So, with the hope of selling both versions to the same high-end customers, the firm may introduce V1 even when $\alpha[\beta v(1+\theta) - c] \geq v - c$.

When we compare the high and low period-1 prices to get the optimal price, we find that the result is very similar to that of Proposition 2.3.2, except that the critical threshold ϕ is different. Each zero-order-mixed REE is with the low price $p_1 = R_f$ and has the profit $\alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$. This profit can be achieved with $p_1 = v$ and $q_1 = 0$, and thus it is dominated by the profit associated with the high price $p_1 = v$. Hence, the zero-order-mixed REE does not occur under dual rollover.

2.4.2 Single Rollover

Under single rollover in period 2, there is only V2 and two groups of customers (s P1B and $(n-s)$ P1NB) to consider. Bargain hunters are in the market, but they cannot afford V2. Therefore, there are only two possible values for p_2 in period 2 – either $\beta v\theta$ or $\beta v(1+\theta)$ – which are the reservation prices for P1B and P1NB, respectively. Corresponding to $p_2 = \beta v(1+\theta)$ and $p_2 = \beta v\theta$, there are two values of period-2 demand, which are $(n-s)$ for P1NB and n for high-end customers (P1B and PINB). This implies a trade-off between pricing low to sell to all high-end customers and pricing high to only sell to P1NB. The firm chooses p_2 based on the market saturation as shown in Proposition 2.4.3.

Proposition 2.4.3 *The firm's period-2 optimal decisions are:*

$p_1 = R_f$		$p_1 = v$		
every ϕ		$\phi \leq \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$		$\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$
$n \leq Aq_1$	$n > Aq_1$	$n \leq Aq_1$	$n > Aq_1$	*all* ns
L	H	L	H	H
$w(q_1, R_f, 1) = \beta v F(Aq_1)$		$w(q_1, v, 0) = \beta v F(Aq_1)$		$w(q_1, v, 0) = 0$

where $A = \frac{\beta v(1+\theta)-c}{\beta v}$, *and* L *denotes* $p_2 = \beta v\theta$ *and* $q_2 = n$ *with* $\Pi_2^S = (\beta v\theta - c)n + \sigma(q_1 - s)$; H *denotes* $p_2 = \beta v(1+\theta)$ *and* $q_2 = n - s$ *with* $\Pi_2^S = [\beta v(1+\theta) - c](n-s) + \sigma(q_1 - s)$.

Recall that with low and medium innovation, the firm gains an absolute pricing power for V2 by committing to single rollover and completely eliminates strategic waiting. However, this is not so with high innovation. Comparing Proposition 2.4.3 with Proposition 2.4.1, when $p_1 = R_f$ or when $p_1 = v$ and $\phi \leq \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, even in the absence of V1 under single rollover, the firm uses the same pricing strategy for V2 as that under dual rollover. Namely, the cannibalization between these two versions is so low with high innovation that the absence of V1 under single rollover *does not strengthen* the firm's pricing power for V2.

Furthermore, when $p_1 = R_f$ or when $p_1 = v$ and $\phi \leq \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, with the same q_1, strategic customers have *the same* expected waiting surpluses under both rollovers. This is different from low and medium innovation cases, in which the firm can reduce the waiting surplus to zero by committing to single rollover. With medium innovation, V2 can be sold to P1NB only, and thus eliminating the cannibalization from V1 by committing to single rollover is equivalent to *committing to* the high price for V2. In contrast, with high innovation, the firm has the chance to induce P1B to replace V1 with V2. If the market is highly saturated, which occurs when $p_1 = R_f$ and $n \leq Aq_1$, or when $p_1 = v$, $\phi \leq \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$ and $n \leq Aq_1$, then it is more profitable to set a low price to induce replacements. This leads to a positive surplus for P1NB. In addition, under dual rollover, although the positive surplus can either be from V2 or V1, V2 is always preferred. Therefore, despite the removal of V1 under single rollover, waiting customers can still obtain the positive surplus from their preferred V2.

When $p_1 = v$ and $\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, the expected waiting surplus is zero. So, all high-end customers want to buy in period 1 at $p_1 = v$. This seems similar to the low and medium innovation cases, in which single rollover can eliminate the waiting surplus completely. However, unfortunately for the firm, as shown in Lemma 6.1.6 in the appendix, this ideal case ($p_1 = v$ and $\chi = 0$) cannot be attained in an REE. Thus, single rollover is not effective in reducing the waiting incentive even with a high proportion of strategic customers.

When we compare the firm' profits with high and low period-1 prices to get the optimal p_1, we find that unlike dual rollover, a zero-order-mixed REE with $p_1 = R_f$ may exist under single rollover. This is because according to Lemma 6.1.6, when $\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, the only possible REE is with $p_1 = R_f$. So, when $\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, a zero-order-mixed REE with $p_1 = R_f$, if any, cannot be dominated by an equilibrium with $p_1 = v$.

2.4.3 Optimal Rollover Strategy

Finally, we compare the performance of single rollover and dual rollover with high innovation. Let $\mathbf{E}[\Pi^{D,h}]$ and $\mathbf{E}[\Pi^{D,l}]$ be the firm's total profit under dual rollover with a period-1 high price and low price, respectively. Similarly, we define $\mathbf{E}[\Pi^{S,h}]$ and $\mathbf{E}[\Pi^{S,l}]$ under single rollover. Let $\mathbf{E}[\Pi^{D*}] = \max\{\mathbf{E}[\Pi^{D,h}], \mathbf{E}[\Pi^{D,l}]\}$ and $\mathbf{E}[\Pi^{S*}] = \max\{\mathbf{E}[\Pi^{S,h}], \mathbf{E}[\Pi^{S,l}]\}$. Consequently, $\mathbf{E}[\Pi^{S*}]$ and $\mathbf{E}[\Pi^{D*}]$ are the firm's REE profits under single and dual rollover, respectively. Let $\phi^H = \inf\{\phi : E[\Pi^{S*}] = E[\Pi^{D*}]$, where $0 \le \phi < 1\}$.

Proposition 2.4.4 (High innovation) *i) If there is an REE under single rollover, then we have the following.*

i.a) If a ϕ^H does not exist, then dual rollover is always better than single rollover.

i.b) If a ϕ^H exists, then single rollover is optimal iff $\phi \ge \phi^H$.

ii) If there is a zero-order-mixed REE under single rollover, then dual rollover is always better than single rollover.

According to Proposition 2.4.4 ii), the firm never needs to implement an ordering strategy suggested by a zero-order-mixed REE. From Section 2.4.2, single rollover does not strengthen the firm's pricing power for V2. In addition, with high innovation, P1NB prefer V2 to V1. Thus, even without V1 under single rollover, P1NB can still get their preferred option V2. Hence, it is not clear how the absence of V1 can improve the firm's profit. Proposition 2.4.5 helps us to understand how single rollover gains an advantage over dual rollover.

Proposition 2.4.5 *Suppose that there is an REE under single rollover. Then we have the following.*

i) If $p_1^* = v$ under single rollover, then dual rollover is always better than single rollover.

ii) The necessary and sufficient condition for the existence of a ϕ^H is that $\mathbf{E}[\Pi^{S,l}] \ge \mathbf{E}[\Pi^{D,l}]$.

Proposition 2.4.5 i) states that single rollover cannot outperform dual rollover if a high period-1 price is optimal, i.e., $p_1^* = v$, under single rollover. Equivalently, $p_1^* = R_f$ under single rollover is a necessary condition for single rollover to be optimal. Proposition 2.4.5 ii) further states that if single rollover can beat dual rollover given that the period-1 price is low in both rollovers ($\mathbf{E}[\Pi^{S,l}] \ge \mathbf{E}[\Pi^{D,l}]$), then there always exists a ϕ^H such that single rollover outperforms dual rollover for $\phi \ge \phi^H$.

Next we explain why $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$ can occur. We find that the two profit functions $\mathbf{E}[\Pi^{S,l}]$ and $\mathbf{E}[\Pi^{D,l}]$ are structurally the same except for that the leftover V1 is sold at the price σ under single rollover and at the price δ under dual rollover in period 2. Since $\sigma < \delta$, for $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$ to occur, the firm must *suffer* from a high disposal value σ, which is counterintuitive. The direct economical benefit of a higher disposal value is a higher revenue from the leftover V1. However, this is only part of the story. With a higher disposal value, the firm tends to order more V1 at the beginning of period 1. This leads to a higher expected sale, and in turn to a more saturated market. Recall that with high innovation, replacement becomes an option. In a saturated market, the firm tends to set a low price for V2 to induce P1B to purchase again. Strategic customers recognize that a higher disposal value leads to a higher possibility of a low price for V2, so they are less willing to purchase early unless p_1 is very low. This is the indirect behavioral impact of a higher disposal value, which adversely affects the firm's profit. When the proportion of strategic customers is high, the behavioral impact dominates the economical benefit, and thus leads to a low profit. This interplay also underscores the importance of joint pricing and stocking level consideration.

2.5 Numerical Illustration

In this section we numerically quantify the value of single rollover over dual rollover. We also study how the innovation θ and the proportion ϕ of strategic customers affect the value of single rollover. Generalizing the notations in Section 2.4.3 for low, medium or high innovations, we use $\mathbf{E}[\Pi^{S*}]$ and $\mathbf{E}[\Pi^{D*}]$ to denote the firm's respective equilibrium profits under single and dual rollover. We define $(\mathbf{E}[\Pi^{S*}] - \mathbf{E}[\Pi^{D*}])/\mathbf{E}[\Pi^{D*}]$ as the value of single rollover compared to dual rollover. We first summarize two main observations from our analysis.

Observation 1: Single rollover is more valuable when the innovation is low or medium.

Observation 2: Single rollover is more valuable when the proportion of strategic customers is high.

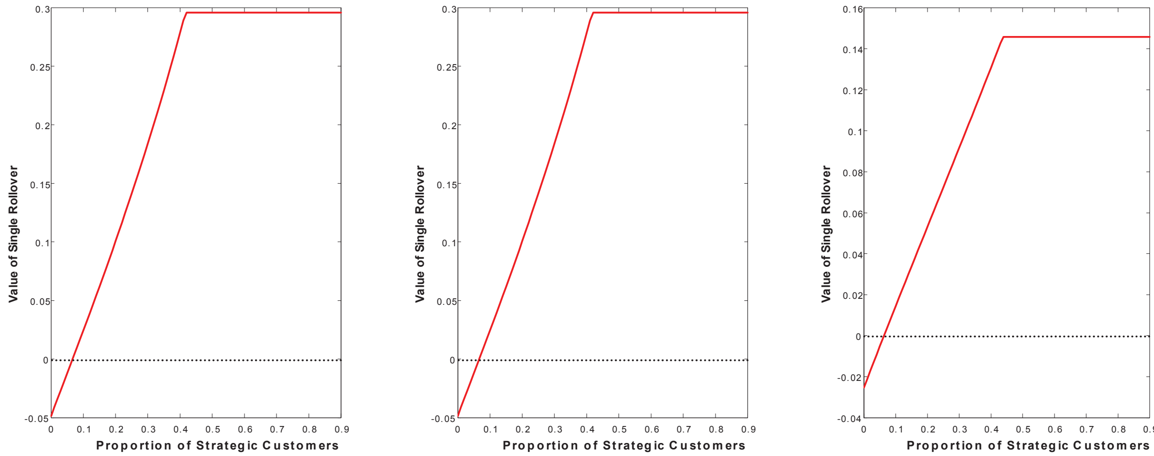


Figure 2.2. Value of Single Rollover. Left to Right: (a) Low Innovation $\theta = 0.2$; (b) Medium Innovation $\theta = 0.6$; (c) High Innovation $\theta = 0.9$

Figures 2.2 (a), (b) and (c) are examples for low, medium and high innovations, respectively. In the examples, $v = 10$, $c = 5$, $\alpha = 0.6$, $\beta = 0.6$, $\delta = 3$, $\sigma = 1$, and N is Gamma distributed with mean 50 and standard deviation 25. From these figures, as the innovation increases, the value of single rollover decreases. In low and medium innovation cases, the profit increase with single rollover can be as much as 29% and 14.6%, respectively. With high innovation, the profit increase is less than 1%, which confirms Observation 1. With high innovation, single rollover neither ensures the firm of a larger pricing power for V2 as the cannibalization is already low, nor completely eliminates the waiting incentive.

In Propositions 2.3.5 and 2.4.4, we prove that single rollover has an advantage over dual rollover when the proportion of strategic customers is greater than a critical value. From Figures 2.2 (a)-(c), we see that with low and medium innovations, the critical value is less than 10%, which is common these days (McWilliams 2004); with high innovation, the critical value is around 70%, which is foreseeable in the future considering customers are better aided by advanced tools, such as technical forums (e.g., Computerworld.com) and online deal forums (e.g., DealSea.com). We obtain similar critical values for the proportion of strategic customers with other parameter combinations.

From Proposition 2.3.5, with low and medium innovations, a necessary condition for single rollover to outperform dual rollover is that the market-disposal spread $(\delta - \sigma)$ be less than a critical value $\triangle$. Our numerical study shows that $\triangle$ can be very large, which implies that single rollover performs better in a wide parameter range. For example, for $\delta = 3$, even when the firm donates V1 with $\sigma = 0$, single rollover does much better than dual rollover (as much as 26% and 14% increase in profit, respectively, for $\theta = 0.2$ and $\theta = 0.6$) as long as the proportion of strategic customers is more than 10%. In summary, our numerical study shows that single rollover can be widely adopted in practice to increase a firm's profit, especially when the innovation is not very high.

2.6 Conclusion

We analyze single and dual rollover strategies in low, medium and high innovation cases, and find that single rollover can improve a firm's profit in all innovation cases under some conditions. Our analytical study provides the rationale for why single rollover can outperform dual rollover in all three innovation cases. Both cannibalization and postponement effects are discussed while comparing the two rollover strategies. We also study the firm's dynamic pricing and inventory decisions over two periods under each rollover strategy, the joint pricing for V1 and V2, and the impact of the strategic behavior on pricing.

In our model, only one stream of high-end customers arrives in period 1 and constitutes markets in both periods. If there is another stream of high-end customers arriving in period 2 and are interested in buying only the new version, two changes may occur. On one hand, under dual rollover the firm has more incentive to price the new version high to target the customers who have not bought in period 1, which implies that single rollover is less effective. On the other hand, however, single rollover may become more important. This is because the larger the number of high-end customers is, the more additional profit can be made with single rollover by keeping the price high. The managerial insights above continue to hold despite these two changes.

CHAPTER 3

IMPACT OF STRATEGIC CUSTOMER BEHAVIOR AND ROLLOVER STRATEGIES ON PRODUCT INNOVATION

3.1 Synopsis

Innovation is one of the most important processes for firms to create new markets, transform industries, and sustain growth. In recent times, we have seen frequent new product introductions and short product life cycles, especially in the high-tech and fashion industries. For example, Apple introduces new iPhone versions annually and the fashion retailers usually bring in new designs at least once per season. However, a higher innovation does not guarantee a higher return or a higher profit. As reported in Arndt and Einhorn (2010), some of the 50 most innovative companies have very low or negative stock returns, revenue growth and/or margin growth. Many factors such as the cost of R&D, the purchasing pattern of customers, the degree of competitiveness in the market, the innovation pace and the way in which a firm rolls over from a product (version) to the next product (version), impact the profitability. We use "version" and "product" interchangeably through this chapter.

When rolling over from an old product to the next, it would be ideal to have the inventory of the old product completely depleted. However, this ideal is difficult to attain in reality with an uncertain demand, and a firm usually needs to deal with the leftover old product when rolling over to a new product. There are two primary product rollover strategies: single rollover and dual rollover. With dual rollover, both the leftover old product and the new product remain and are sold in the market. With single rollover, the leftover old product is disposed of outside the market. Fire sales, dismantling products for spare parts, recycling the material for future use, selling leftovers in the overseas markets, at discount stores or through secondary channels, or donating them for charity are common ways to dispose of leftover inventory. See Tibben-Lembke (2004) for additional approaches to disposing of the unsold inventory.

A firm usually receives higher revenue from the leftovers under dual rollover than under single rollover. However, dual rollover can backfire when customers behave more strategically. Aided by advanced tools, such as technical fora (e.g., Computerworld.com and Zdnet.com) and online deal fora (e.g., DealSea.com), customers can anticipate introductions of new products. Therefore, instead of purchasing a current product, they may wait for the release of an upgraded new product, or for the markdown of the current product if the firm decides to keep its leftovers in the market. Thus, strategic customers are more willing to delay their purchase when anticipating a possible markdown of the current product under dual rollover. This strategic waiting behavior has greatly hurt a firm's profit in practice (McWilliams 2004) and has received much attention lately in academia.

An interesting question is: how do the *demand-side* (strategic waiting behavior) effect and *channel-side* (product rollover strategies) effect jointly impact a firm's optimal innovation pace and profit? To answer this, we study a firm's optimal profit and innovation pace decisions in four settings: when customers are myopic (i.e., they do not consider the waiting option) under single or dual rollover, or when the customers are strategic (i.e., they consider the waiting option) under single or dual rollover. The market demand is assumed to be uncertain. Within each scenario, the firm makes decisions for the innovation level, the prices and the quantities for both the old and new products. The innovation level leads to an R&D expenditure. Thus, in addition to demand-side and channel-side effects, we also consider the *supply-side* (R&D cost) effect.

Our work is related to three streams of literature: product rollover strategies, product innovation/quality decisions over time, and strategic customer behavior in operations management (OM). Despite their importance, product rollover strategies have not received enough attention in academia. Billington et al. (1998) and Erhun et al. (2007) provide hands-on experience and managerial insights. To our knowledge, there are only six papers that compare different rollover strategies analytically: Levinthal and Purohit (1989), Lim and Tang

(2006), Ferguson and Koenigsberg (2007), Arslan et al. (2009), Koca et al. (2010), and Liang et al. (2011a). In all of these, the innovation/quality level is exogenous, so they do not answer our primary research question of how the innovation level varies in the different settings. In addition, most of them focus on the cannibalization between products and/or the product introduction and phase-out times, and they do not study the strategic waiting behavior under different rollover strategies.

Product innovation/quality decisions in the presence of strategic customers have been studied in the marketing and economics literatures. Pricing (Kornish 2001), trade-in and buyback (Fudenberg and Tirole 1998), planned obsolescence (Fishman and Rob 2000) are some of the solutions to induce strategic customers to purchase early. Moorthy and Png (1992) study whether to release products simultaneously or sequentially. Toktay et al. (2011) consider product introduction strategies with exclusivity-seeking behavior. Krishnan and Ramachandran (2011) study how to use a modular upgradable architecture to alleviate consumer concerns about product obsolescence. We refer to Shane and Ulrich (2004) for a detailed review of innovation and product development. Our study departs from this stream of literature by studying how rollover strategies and strategic waiting behavior jointly impact the innovation decision.

The most related papers in the OM literature are Su and Zhang (2008), Cachon and Swinney (2009), and Lai et al. (2010), where strategic customers face the tradeoff between buying the product at full price in the regular selling season and buying it at a marked-down price in the market-clearing season with the risk of not being able to get it due to stockout. These three papers study different mechanisms for strategic waiting mitigation when the demand is uncertain and the inventory level is a decision variable: quantity and price commitments in Su and Zhang (2008), quick response strategy in Cachon and Swinney (2009), and posterior price matching in Lai et al. (2010). Our model studies how another mechanism – single rollover – can be used to mitigate the strategic customers' waiting incentive. Considering only one product, these three papers and others (Su and Zhang 2009, Su 2008, Cachon and Swinney 2011, Liu and van Ryzin 2008, Aviv and Pazgal 2008, Yin et al. 2009, Prasad et al. 2010, and Huang and Van Mieghem 2010) do not study the product introduction and innovation decisions for the new product. While Liang et al. (2011a) study product rollover and product introduction, their focus is on the performance of single and dual rollover under different market compositions (based on the proportion of strategic and myopic customers) and customer purchasing behaviors (one-time and repeat purchase). More importantly, the innovation level is exogenous in Liang et al. (2011a).

We show analytically that the strategic waiting behavior accelerates the innovation process – *a counterintuitive finding*. Conventional wisdom is that a firm has a lower pricing power with strategic customers than with myopic customers, and this reduces the return on innovation investment and in turn incentive to innovate. Dhebar (1994) supports this conventional wisdom by showing that strategic customer behavior imposes a *demand-side* constraint on the rate of product improvement. Our study, however, shows the opposite by observing that a firm should produce less of the old product when facing strategic customers, and should produce even less when innovating more. This observation leads to two drivers of our surprising result. First, a firm producing less of the old product has more unsatisfied customers who can buy the new product. Thus, a firm facing strategic customers has more incentive to innovate in order to charge a higher price for the new product. Second, anticipating the lower chance to get the leftover old product due to a lower production level, strategic customers are more willing to buy the old product early rather than to wait for a markdown. A firm can increase this willingness with more innovation, which results in an even lower production level and that allows the firm to charge a higher price for the old product.

Another *interesting* result we find is that when customers are myopic, single rollover rather than dual rollover speeds up the innovation process. With both products in the market under dual rollover, people usually think that the firm should innovate more to reduce cannibalization. However, our study shows the opposite. This is because with a lower revenue from the leftovers under single rollover compared to dual rollover, the firm prefers to produce less to reduce the overstocking loss resulting from the demand uncertainty.

This leads to more unsatisfied customers and, in turn, to a higher incentive for innovation.

We also find that when customers are myopic, the firm always provides a more innovative product but earns a lower profit under single rollover compared to dual rollover, and the innovation level and profit cannot be improved simultaneously for any selected rollover strategy. In contrast, when customers are strategic, the firm can provide a more innovative product while earning a higher profit by adopting the appropriate rollover strategy compared to the other rollover strategy. This underscores the importance of choosing the appropriate rollover strategy when customers are strategic.

3.2 Model Description

To assess the impact of product rollover strategies and strategic customer behavior on a firm's innovation level and profit, we analyze the equilibrium between the firm and the customers in four different settings as summarized in Table 3.1. These four settings are discussed in Sections 3.3.1-3.3.2 and Sections 3.4.1-3.4.2.

Table 3.1. Four Settings of Customer Types and Rollover Strategies

Myopic customers under Dual Rollover (M-DR)	Myopic customers under Single Rollover (M-SR)
Strategic customers under Dual Rollover (S-DR)	Strategic customers under Single Rollover (S-SR)

Both myopic and strategic customers are high-end customers, and they arrive in period 1. High-end customers' valuations of V1 and V2 are v and $v(1+\theta)$, respectively, where θ is the additional innovation of V2 over V1. For example, θ can represent the number of new features added to V2. Strategic customers decide whether to purchase V1 in period 1 with the waiting option in mind, while myopic customers make the decision without considering the waiting option. Similar to Dhebar (1994) and Moorthy and Png (1992), we assume (1) a high-end customer arriving in period 1 will remain in the market in period 2 unless he gets one unit of V1 in period 1, and (2) there is no second-hand market.

Another stream of high-end customers arrive in period 2. They are attracted by V2, and thus they are interested in purchasing only V2. We can describe high-end customers arriving in period 1 as functional-product-seeking customers, and those arriving in period 2 as innovative-product-seeking customers. For simplicity, we term high-end customers arriving in period 2 as *newcomers*, and use the term *high-end customers* only for those arriving in period 1. The market size for high-end customers is a random variable $N > 0$ with c.d.f. $F(\cdot)$ and density $f(\cdot)$, while the market size for newcomers is a random variable $K(\theta) > 0$ with c.d.f. $G_\theta(\cdot)$ and density $g_\theta(\cdot)$, where $K(\theta)$ increases in θ stochastically. When both the newcomers and high-end customers remaining in the market want V2 in period 2, we assume that remaining high-end customers have purchase priority over newcomers. We also assume that customers within each segment have equal priority. A similar assumption is used in Su and Zhang (2008), Cachon and Swinney (2009), and Lai et al. (2010).

Besides high-end customers, there are unlimited bargain-hunters or low-end customers. They value the product so low that they only buy it on sale. So, they arrive in period 2 and buy only the leftover V1, if any, under dual rollover when its marked-down price is not larger than their valuation δ.

Under each scenario, a profit-maximizing firm may introduce a product V1 in period 1 and its upgraded version V2 in period 2. Prior to period 1, the firm decides the innovation level $\theta \geq 0$ for V2, which incurs an expenditure $I(\theta)$ for R&D. The unit production cost c is the same for both versions. This is reasonable particularly for high-tech and fashion products. For example, "iPad 2's 32-gigabyte model with a GSM/HSPA air standard carries a bill of materials totaling $326.60, while the 32-gigabyte version equipped with a CDMA air standard has a materials bill of $323.25, according to IHS iSuppli. That compares with a $320 bill of materials for the first-generation 32-gigabyte iPad..."(Becker 2011). In both periods, the firm decides the prices and production levels of V1 and V2 as detailed below. Under single rollover, the firm receives the disposal value $\sigma \geq 0$ from each unit of the leftover V1. We assume $\sigma, \delta < c$, to avoid producing an infinite amount. Note that the firm will not charge a price lower than δ for the leftovers. We also assume $\sigma < \delta$ as the disposal value of the leftover V1 under single rollover is usually lower than its selling price in the market. We refer to $(\delta - \sigma)$ as the *market-disposal spread*. The inventory remaining at the end of period 2 has zero value. The firm discounts its profit in period 2 by a factor of α, $0 < \alpha < 1$. The firm knows v, σ, δ, c, $F(\cdot)$ and $G_\theta(\cdot)$, and observes the realized high-end market size at the end of period 1. Customers observe the prices but not the production levels.

Below is a summary of the event sequence in each of the four settings:

1. The firm decides the innovation level θ at a cost of $I(\theta)$.

2. Period 1 begins. The firm decides the price p_1 and the production level q_1 for V1, and high-end customers (strategic or myopic customers) decide whether to buy or not. The high-end market size N is realized at the end of period 1.

3. Period 2 begins. Under dual rollover, the firm decides the price p_2, the production level q_2 for V2, and the marked-down price p_1' for the leftover V1, if any. Under single rollover, it decides p_2 and q_2 for V2, and receives σ for each unit of the leftover V1, if any. High-end customers who have not purchased V1, newcomers and all low-end customers make purchase decisions. Any leftovers at the end of period 2 have zero value.

We assume the following reasonable properties for the innovation cost $I(\theta)$: $I(0) = 0$, $I'(\theta) > 0$ and $I''(\theta) > 0$. Let $\theta_{\max} = \frac{(1-\alpha)(v-c)}{\alpha v}$. We will show later (in Sections 3.3.1, 3.3.2, 3.4.1 and 3.4.2) that in the case with $\theta \geq \theta_{\max}$, the firm skips V1 ($q_1 = 0$) and introduces only V2, and thus there is no difference between single and dual rollovers, or between myopic and strategic customers. In this chapter, "higher (resp., lower)" means "not lower (resp., not higher)", and "more (resp., less)" means "not less (not more)".

3.3 Myopic Customers

3.3.1 Dual Rollover (M-DR)

In period 1, the firm decides p_1 and q_1 targeting myopic customers, the only segment in the market, which means the expected sale in period 1 is $\mathbf{E}_N[\min\{N, q_1\}]$. Two possibilities occur after the high-end market size N is realized at the end of period 1.

(1) If $q_1 > N$, then all myopic customers are satisfied and there are $q_1 - N$ units of V1 left. Thus in period 2, the firm will set $p_2 = v(1+\theta)$ and sell V2 to the newcomers, and it will set $p_1' = \delta$ to clear the leftover V1.

(2) If $q_1 < N$, then there are no leftover V1 and there are $N - q_1$ myopic customers still in the market; in period 2, the firm will set $p_2 = v(1+\theta)$ and sell V2 to the remaining myopic customers and the newcomers.

Let $(x)^+$ denote $\max(x, 0)$. Combining (1) and (2), the firm's period-2 optimal profit is

$$\pi_2(q_1, \theta) = \mathbf{E}_N\{\max_{q_2} [v(1+\theta)\mathbf{E}_{K(\theta)}[\min\{(N-q_1)^+ + K(\theta), q_2\}] - cq_2] + \delta(q_1 - N)^+\}. \quad (3.1)$$

Clearly, in period 2, the firm faces a newsvendor problem with demand $(N - q_1)^+ + K(\theta)$, ordering cost c and selling price $v(1+\theta)$. Because N is realized before period 2 starts, the firm will produce $q_2 = (N - q_1)^+ + G_\theta^{-1}(\frac{v(1+\theta)-c}{v(1+\theta)})$ to sell V2 to every remaining myopic customer and some of the newcomers (following the newsvendor critical fractile). After substituting q_2 into (3.1) and reorganizing terms, we have

$$\pi_2(q_1, \theta) = H(\theta) + [v(1+\theta) - c]\mathbf{E}_N[N - q_1]^+ + \delta\mathbf{E}_N[q_1 - N]^+,$$

where $H(\theta) = v(1+\theta)\mathbf{E}_{K(\theta)}[\min\{K(\theta), G_\theta^{-1}(\frac{v(1+\theta)-c}{v(1+\theta)})\}] - cG_\theta^{-1}(\frac{v(1+\theta)-c}{v(1+\theta)})$. Notice that $H(\theta)$ is the optimal profit of the newsvendor problem selling to newcomers $\max_x v(1+\theta)\mathbf{E}_{K(\theta)}[\min\{K(\theta), x\}] - cx$. Because $v(1+\theta)$ increases in θ and $K(\theta)$ increases in θ stochastically, we know that $H(\theta)$ increases in θ. That is, with a higher θ, the firm can cater to more newcomers and charge a higher price to each, and thus it earns a higher profit from newcomers. The second and third term in $\pi_2(q_1, \theta)$ represent the expected profit from selling V2 to the unsatisfied myopic customers and the expected revenue from selling the leftover V1 to the low-end customers, respectively. The analysis in period 2 is similar for all four settings in Table 1.

Without the waiting option in mind, myopic customers buy V1 in period 1 if $p_1 \leq v$. So for any given θ, the firm sets $p_1 = v$ and determines the optimal q_1 to maximize its expected two-period profit as

$$\begin{aligned}\pi^{\text{M-DR}}(q_1,\theta) &= v\mathbf{E}_N[\min\{N,q_1\}] - cq_1 + \alpha\pi_2(q_1,\theta)\\ &= \{v - \alpha[v(1+\theta)-c] - \alpha\delta\}\mathbf{E}_N[\min\{N,q_1\}]\\ &\quad - (c-\alpha\delta)q_1 + \alpha H(\theta) + \alpha[v(1+\theta)-c]\mathbf{E}_N[N],\end{aligned}$$

where the superscript M-DR means the setting when the high-end customers are myopic and the dual rollover is adopted. Notice that the last two terms are independent of q_1. For a given $\theta < \theta_{\max}$ (i.e., $v > c + \alpha[v(1+\theta)-c]$), the optimal order quantity can be found from the newsvendor critical fractile as

$$q_1^{\text{M-DR}}(\theta) = F^{-1}\left(\frac{v - c - \alpha[v(1+\theta)-c]}{v - \alpha[v(1+\theta)-c] - \alpha\delta}\right); \tag{3.2}$$

for $\theta \geq \theta_{\max}$, $q_1^{\text{M-DR}}(\theta) = 0$. That is, when the discounted product margin of selling V2 dominates that of selling V1 (i.e., $v - c \leq \alpha[v(1+\theta)-c]$ or $\theta \geq \theta_{\max}$), the firm prefers to keep the market for V2 by skipping V1.

Let $\Pi^{\text{M-DR}}$ and $\Pi^{\text{M-DR}}(\theta)$ denote the *firm's optimal total expected profit* and the *firm's total expected profit function* when selling to myopic customers under dual rollover, respectively. Similar naming conventions are used later. Then, the optimal innovation level $\theta^{\text{M-DR}}$ can be found by maximizing the profit:

$$\Pi^{\text{M-DR}} = \max_{\theta\geq 0}\Pi^{\text{M-DR}}(\theta) = \max_{\theta\geq 0}[\pi^{\text{M-DR}}(q_1^{\text{M-DR}}(\theta),\theta) - I(\theta)]. \tag{3.3}$$

Notice that $q_1^{\text{M-DR}}(\theta)$ either solves $\frac{\partial\pi^{\text{M-DR}}(q_1,\theta)}{\partial q_1} = 0$ or is a constant zero. We have

$$\begin{aligned}\frac{d\Pi^{\text{M-DR}}(\theta)}{d\theta} &= \frac{\partial\pi^{\text{M-DR}}(q_1^{\text{M-DR}}(\theta),\theta)}{\partial\theta} - \frac{dI(\theta)}{d\theta}\\ &= \alpha v\mathbf{E}_N[N - q_1^{\text{M-DR}}(\theta)]^+ + \alpha H'(\theta) - I'(\theta).\end{aligned} \tag{3.4}$$

Clearly, a higher θ impacts $\Pi^{\text{M-DR}}(\theta)$ favorably by increasing the price that can be charged for V2 and attracting more newcomers, and adversely by incurring a higher R&D expenditure. Lemma 3.3.1 characterizes the optimal innovation level $\theta^{\text{M-DR}}$.

Lemma 3.3.1 (M-DR) *The optimal innovation level* $\theta^{\text{M-DR}}$ *is either zero or it satisfies* $\alpha v \mathbf{E}_N[N - q_1^{\text{M-DR}}(\theta)]^+ + \alpha H'(\theta) - I'(\theta) = 0$*, and the optimal order quantity is either zero or given by* (3.2)*.*

3.3.2 Single Rollover (M-SR)

The analysis for single rollover is the same as that for dual rollover except in period 2, when $q_1 > N$, instead of selling the leftover V1 at $p_1' = \delta$ in the market, the firm disposes of all leftover V1 outside of the market and receives σ for each unit.

Lemma 3.3.1 does not establish the unimodal property of $\Pi^{\text{M-DR}}(\theta)$ or the uniqueness of the solution for $\frac{d\Pi^{\text{M-DR}}(\theta)}{d\theta} = 0$, which turns out to greatly depend on the properties of $I(\theta)$. Without the unimodal property or uniqueness, however, we can still compare the optimal innovation level $\theta^{\text{M-SR}}$ and the firm's optimal total expected profit $\Pi^{\text{M-SR}}$ under single rollover with those under dual rollover as summarized in Proposition 3.3.2.

Proposition 3.3.2 (M-SR vs. M-DR) *When high-end customers are myopic, the firm earns a higher profit but provides a less innovative V2 under dual rollover compared to single rollover. That is,* $\Pi^{\text{M-DR}} \geq \Pi^{\text{M-SR}}$ *and* $\theta^{\text{M-DR}} \leq \theta^{\text{M-SR}}$*.*

Proof. We can see that

$$\begin{aligned}\Pi^{\text{M-DR}} &= \pi^{\text{M-DR}}(q_1^{\text{M-DR}}(\theta^{\text{M-DR}}), \theta^{\text{M-DR}}) - I(\theta^{\text{M-DR}}) \\ &\geq \pi^{\text{M-DR}}(q_1^{\text{M-SR}}(\theta^{\text{M-SR}}), \theta^{\text{M-SR}}) - I(\theta^{\text{M-SR}}) \\ &\geq \pi^{\text{M-SR}}(q_1^{\text{M-SR}}(\theta^{\text{M-SR}}), \theta^{\text{M-SR}}) - I(\theta^{\text{M-SR}}) \\ &= \Pi^{\text{M-SR}},\end{aligned}$$

where the first inequality is from the fact that $\Pi^{\text{M-DR}}(\theta)$ is maximized at $\theta = \theta^{\text{M-DR}}$ and the second inequality is due to

$$\pi^{\text{M-SR}}(q_1, \theta) = v\mathbf{E}_N[\min\{N, q_1\}] - cq_1 + \alpha\{H(\theta) + [v(1+\theta) - c]\mathbf{E}_N[N - q_1]^+ + \sigma\mathbf{E}_N[q_1 - N]^+\}$$

and $\sigma < \delta$.

Next we prove $\theta^{\text{M-DR}} \leq \theta^{\text{M-SR}}$. Let $\Pi^{\text{M-SR}}(\theta) = \pi^{\text{M-SR}}(q_1^{\text{M-SR}}(\theta), \theta) - I(\theta)$. Similar to (3.4), we have

$$\frac{d\Pi^{\text{M-SR}}(\theta)}{d\theta} = \alpha v\mathbf{E}_N[N - q_1^{\text{M-SR}}(\theta)]^+ + \alpha H'(\theta) - I'(\theta). \tag{3.5}$$

Since

$$q_1^{\text{M-DR}}(\theta) = F^{-1}\left(\frac{v - c - \alpha[v(1+\theta) - c]}{v - \alpha[v(1+\theta) - c] - \alpha\delta}\right) > F^{-1}\left(\frac{v - c - \alpha[v(1+\theta) - c]}{v - \alpha[v(1+\theta) - c] - \alpha\sigma}\right) = q_1^{\text{M-SR}}(\theta)$$

for $\theta < \theta_{\max}$ and $q_1^{\text{M-DR}}(\theta) = q_1^{\text{M-SR}}(\theta) = 0$ for $\theta \geq \theta_{\max}$, we have $q_1^{\text{M-DR}}(\theta) \geq q_1^{\text{M-SR}}(\theta)$. This inequality together with (3.4) and (3.5) imply $\frac{d\Pi^{\text{M-DR}}(\theta)}{d\theta} \leq \frac{d\Pi^{\text{M-SR}}(\theta)}{d\theta}$ for any θ. Note for any $\theta \geq \theta^{\text{M-SR}}$,

$$\int_{\theta^{\text{M-SR}}}^{\theta} \frac{d\Pi^{\text{M-DR}}(\theta)}{d\theta} d\theta \leq \int_{\theta^{\text{M-SR}}}^{\theta} \frac{d\Pi^{\text{M-SR}}(\theta)}{d\theta} d\theta$$

$$\Longrightarrow \Pi^{\text{M-DR}}(\theta) - \Pi^{\text{M-DR}}(\theta^{\text{M-SR}}) \leq \Pi^{\text{M-SR}}(\theta) - \Pi^{\text{M-SR}}(\theta^{\text{M-SR}})$$

$$\Longrightarrow \Pi^{\text{M-DR}}(\theta) + \Pi^{\text{M-SR}}(\theta^{\text{M-SR}}) - \Pi^{\text{M-SR}}(\theta) \leq \Pi^{\text{M-DR}}(\theta^{\text{M-SR}})$$

$$\Longrightarrow \Pi^{\text{M-DR}}(\theta) \leq \Pi^{\text{M-DR}}(\theta^{\text{M-SR}}) \text{ because } \Pi^{\text{M-SR}}(\theta^{\text{M-SR}}) - \Pi^{\text{M-SR}}(\theta) \geq 0.$$

Hence, we have $\Pi^{\text{M-DR}}(\theta) \leq \Pi^{\text{M-DR}}(\theta^{\text{M-SR}})$ for any $\theta \geq \theta^{\text{M-SR}}$, and thus $\theta^{\text{M-DR}} \leq \theta^{\text{M-SR}}$. ■

Proposition 3.3.2 shows that when customers are myopic, although single rollover when compared to dual rollover, reduces a firm's profit due to the low disposal value for the leftover V1, it accelerates the innovation process. This is because, with a low disposal value under single rollover, the firm tends to produce less V1 to reduce possible losses from overstocking (that is, $q_1^{\text{M-SR}}(\theta) \leq q_1^{\text{M-DR}}(\theta)$ as shown in the proof of Proposition 3.3.2), and this leads to lower sales on average in period 1 and then a less saturated market for V2 (i.e., more high-end customers remain in the market). Anticipating more high-end customers remaining in the market, the firm has more incentive to invest in R&D in order to charge a higher price for V2. Another way to understand Proposition 3.3.2 is that, when the firm sells the leftover V1 in the market under dual rollover rather than disposes of it outside of the market under single rollover, it expects more revenues from V1 and thus has less motivation to innovate. This is an *interesting* result since with both products in the market under dual rollover, people usually think that the firm should innovate more in order to better differentiate them.

3.4 Strategic Customers

3.4.1 Dual Rollover (S-DR)

Strategic customers are forward-looking and consider the option of buying in period 2 when deciding to purchase or not in period 1. Given an innovation level θ, we adopt the rational expectation concept to model the interaction between the firm and strategic customers. We assume that all strategic customers are homogenous and each of them holds a belief of the *waiting surplus* W_c – the surplus that every strategic customer can get by waiting. The firm holds a belief R_f of the *strategic customers' reservation price* for V1 in period 1. W_c and R_f are privately formed by each strategic customer and the firm, respectively, and thus are not revealed to the others.

The firm should set $p_1 = R_f$ and $q_1(p_1, \theta) = \arg\max_{q_1} \pi^{\text{S-DR}}(p_1, q_1, \theta)$ in order to maximize its expected two-period profit $\pi^{\text{S-DR}}(p_1, q_1, \theta)$ for a given θ. Strategic customers buy V1 in period 1 only when the surplus of buying now is at least the waiting surplus, i.e., $v - p_1 \geq W_c$, which means that their reservation price for V1 in period 1 is $v - W_c$.

In a rational expectation equilibrium, beliefs should be consistent with their outcomes. Therefore, the firm's belief of the strategic customers' reservation price for V1 should be equal to the reservation price from the strategic customers' point of view (i.e., $R_f = v - W_c$), and thus all strategic customers want to buy V1 in period 1. Next, we calculate the *actual* waiting surplus for an individual strategic customer if he decides to wait rather than buy in period 1. W_c should be consistent with the actual waiting surplus. As analyzed in Section 3.3.1, if $q_1 < N$, then there is no V1 left in period 2 and the firm sets $p_2 = v(1+\theta)$, which implies a zero surplus from waiting. A positive waiting surplus can occur only if $q_1 > N$, in which case the firm sets $p_1' = \delta < v$ to clear the leftover V1. Since bargain hunters want to buy the leftover V1 at the low price of δ, the individual waiting strategic customer cannot necessarily get one unit of the leftover V1 and obtain the surplus $v-\ \delta$ when $q_1 > N$. In our model, we follow the efficient rationing assumption as in Su and Zhang (2008), Cachon and Swinney (2009) and Lai et al. (2010): strategic customers have purchase priority over bargain hunters. With this assumption, the waiting customer can certainly get one unit of the leftover V1 in period 2 when $q_1 > N$. Therefore, the actual waiting surplus is $(v-\delta)F(q_1)$ as implied from the above discussion. Clearly, the actual waiting surplus increases in q_1.

We here summarize the rational expectation equilibrium conditions between the firm and strategic customers for a given θ: (i) $p_1 = R_f$, (ii) $q_1(p_1, \theta) = \arg\max_{q_1} \pi^{\text{S-DR}}(q_1, p_1, \theta)$, (iii) $R_f = v - W_c$ and (iv) $W_c = (v-\delta)F(q_1)$. Lemma 3.4.1 characterizes the equilibrium.

Lemma 3.4.1 (S-DR) *i) For any given innovation level θ, there is a unique rational expectation equilibrium between the firm and strategic customers.*
ii) With $\theta < \theta_{\max}$, ii.a) the price $p_1^{\text{S-DR}}(\theta) > c + \alpha[v(1+\theta) - c]$ and the order quantity $q_1^{\text{S-DR}}(\theta) > 0$, and they can be uniquely determined from the two equations: $p_1 = v - (v-\delta)F(q_1)$ and $F(q_1) = \frac{p_1 - c - \alpha[v(1+\theta)-c]}{p_1 - \alpha[v(1+\theta)-c] - \alpha\delta}$, and ii.b) $q_1^{\text{S-DR}}(\theta)$ decreases while $p_1^{\text{S-DR}}(\theta)$ increases in θ.
iii) With $\theta \geq \theta_{\max}$, $q_1^{\text{S-DR}}(\theta) = 0$ and $p_1^{\text{S-DR}}(\theta) = v$.

Proof. The proofs of i) and ii.a) are similar to those of Lemma 2 in Liang et al. (2011a), and details are in Chapter 6. When $q_1^{S-DR}(\theta) = 0$, the value of $p_1^{S-DR}(\theta)$ is of no consequence in practice. But theoretically, we need $p_1^{S-DR}(\theta) = v$ to satisfy REE conditions. ■

In Lemma 3.4.1 ii), it is interesting to see how $q_1^{\text{S-DR}}(\theta)$ and $p_1^{\text{S-DR}}(\theta)$ change as θ increases. With a higher θ and then a higher chargeable price for V2, the firm prefers to limit the sale of V1. So, it reduces the production level $q_1^{\text{S-DR}}(\theta)$ to keep the market for V2. The lower production level of V1, in turn, increases the chance of a stockout and thus reduces the probability of getting the leftover V1 by waiting. Anticipating the lower likelihood of getting the leftover V1 by waiting, strategic customers are willing to pay more for V1 in period 1. Therefore, a higher θ can increase not only the price for V2 but also the price for V1 in period 1.

Similar to (3.3), the optimal innovation level $\theta^{\text{S-DR}}$ can be found by maximizing the profit:

$$\Pi^{\text{S-DR}} = \max_{\theta\geq 0} \Pi^{\text{S-DR}}(\theta) = \max_{\theta\geq 0}[\pi^{\text{S-DR}}(q_1^{\text{S-DR}}(\theta), p_1^{\text{S-DR}}(\theta), \theta) - I(\theta)],$$

where

$$\pi^{\text{S-DR}}(q_1, p_1, \theta) = p_1 \mathbf{E}_N[\min\{N, q_1\}] - cq_1 + \alpha\{H(\theta) + [v(1+\theta) - c]\mathbf{E}_N[N - q_1]^+ + \delta \mathbf{E}_N[q_1 - N]^+\}. \tag{3.6}$$

Note that

$$\begin{aligned}
&\frac{d\Pi^{\text{S-DR}}(\theta)}{d\theta} \\
&= \underbrace{\frac{\partial \pi^{\text{S-DR}}(q_1^{\text{S-DR}}(\theta), p_1, \theta)}{\partial p_1}\bigg|_{p_1 = p_1^{\text{S-DR}}(\theta)} \cdot \frac{dp_1^{\text{S-DR}}(\theta)}{d\theta}}_{\text{Indirect Positive Behavioral Impact}} + \underbrace{\frac{\partial \pi^{\text{S-DR}}(q_1^{\text{S-DR}}(\theta), p_1^{\text{S-DR}}(\theta), \theta)}{\partial \theta}}_{\text{Direct Positive Economic Impact}} - \frac{dI(\theta)}{d\theta} \\
&= \mathbf{E}_N[\min\{N, q_1^{\text{S-DR}}(\theta)\}] \cdot \frac{dp_1^{\text{S-DR}}(\theta)}{d\theta} + \alpha v \mathbf{E}_N[N - q_1^{\text{S-DR}}(\theta)]^+ + \alpha H'(\theta) - I'(\theta).
\end{aligned} \tag{3.7}$$

Comparing (3.4) and (3.7), we can see that the *direct* economic benefit of a higher innovation level exists with either myopic or strategic customers, and the direct impact is via both a higher chargeable price for V2 and more newcomers. When selling to strategic customers, however, in addition to the direct positive economic impact, there is an *indirect* impact on the strategic customers' waiting incentive and thus on the price $p_1^{\text{S-DR}}(\theta)$ of V1 in period 1, as in the discussion for Lemma 3.4.1. Because $\frac{dp_1^{\text{S-DR}}(\theta)}{d\theta} \geq 0$ from Lemma 3.4.1, the indirect behavioral impact favors the firm's profit. Although we do not have a closed-form expression for $\Pi^{\text{S-DR}}$ or $\theta^{\text{S-DR}}$, we can compare them with $\Pi^{\text{M-DR}}$ and $\theta^{\text{M-DR}}$ as in Proposition 3.4.2.

Proposition 3.4.2 (M-DR vs. S-DR) *Under dual rollover, the firm earns a higher profit but provides a less innovative V2 when selling to myopic customers than when selling to strategic customers. That is,* $\Pi^{\text{M-DR}} \geq \Pi^{\text{S-DR}}$ *and* $\theta^{\text{M-DR}} \leq \theta^{\text{S-DR}}$*.*

Proof. We can see that

$$\begin{aligned}\Pi^{\text{M-DR}} &= \pi^{\text{M-DR}}(q_1^{\text{M-DR}}(\theta^{\text{M-DR}}), \theta^{\text{M-DR}}) - I(\theta^{\text{M-DR}})\\ &\geq \pi^{\text{M-DR}}(q_1^{\text{S-DR}}(\theta^{\text{S-DR}}), \theta^{\text{S-DR}}) - I(\theta^{\text{S-DR}})\\ &\geq \pi^{\text{S-DR}}(q_1^{\text{S-DR}}(\theta^{\text{S-DR}}), p_1^{\text{S-DR}}(\theta^{\text{S-DR}}), \theta^{\text{S-DR}}) - I(\theta^{\text{S-DR}})\\ &= \Pi^{\text{S-DR}}.\end{aligned}$$

The last inequality is from $\pi^{\text{M-DR}}(q_1^{\text{S-DR}}(\theta^{\text{S-DR}}), \theta^{\text{S-DR}}) \geq \pi^{\text{S-DR}}(q_1^{\text{S-DR}}(\theta^{\text{S-DR}}), p_1^{\text{S-DR}}(\theta^{\text{S-DR}}), \theta^{\text{S-DR}})$, which is due to $p_1^{\text{S-DR}}(\theta^{\text{S-DR}}) \leq v$.

Next we prove $\theta^{\text{M-DR}} \leq \theta^{\text{S-DR}}$. For $\theta < \theta_{\max}$,

$$q_1^{\text{S-DR}}(\theta) = F^{-1}\left(\frac{p_1^{\text{S-DR}}(\theta) - c - \alpha[v(1+\theta) - c]}{p_1^{\text{S-DR}}(\theta) - \alpha[v(1+\theta) - c] - \alpha\delta}\right) < F^{-1}\left(\frac{v - c - \alpha[v(1+\theta) - c]}{v - \alpha[v(1+\theta) - c] - \alpha\delta}\right) = q_1^{\text{M-DR}}(\theta)$$

due to $p_1^{\text{S-DR}}(\theta) < v$; for $\theta \geq \theta_{\max}$, we have $q_1^{\text{S-DR}}(\theta) = q_1^{\text{M-DR}}(\theta) = 0$. Therefore, $q_1^{\text{S-DR}}(\theta) \leq q_1^{\text{M-DR}}(\theta)$ for any θ. Note that

$$\begin{aligned}\frac{d\Pi^{\text{M-DR}}(\theta)}{d\theta} &= \alpha v \mathbf{E}_N[N - q_1^{\text{M-DR}}(\theta)]^+ + \alpha H'(\theta) - I'(\theta)\\ &\leq \mathbf{E}_N[\min\{N, q_1^{\text{S-DR}}(\theta)\}] \cdot \frac{dp_1^{\text{S-DR}}(\theta)}{d\theta} + \alpha v \mathbf{E}_N[N - q_1^{\text{S-DR}}(\theta)]^+ + \alpha H'(\theta) - I'(\theta)\\ &= \frac{d\Pi^{\text{S-DR}}(\theta)}{d\theta},\end{aligned}$$

where the inequality is due to $\frac{dp_1^{\text{S-DR}}(\theta)}{d\theta} \geq 0$ from Lemma 3.4.1 and $q_1^{\text{S-DR}}(\theta) \leq q_1^{\text{M-DR}}(\theta)$.

With $\frac{d\Pi^{\text{M-DR}}(\theta)}{d\theta} \leq \frac{d\Pi^{\text{S-DR}}(\theta)}{d\theta}$, we can prove the result in a way similar to that in Proposition 3.3.2. ■

Proposition 3.4.2 states that strategic behavior hurts the firm's profit, but speeds up the innovation process which is counter to intuition. Because it is difficult to extract all of the utility from strategic customers, the return on innovation investment when selling to strategic customers is lower compared to selling to myopic customers. Therefore, people usually think that a firm has less incentive to innovate when customers are strategic. Our model, however, *shows the opposite*. There are two reasons for this counterintuitive result. First, the firm produces less V1 to reduce the strategic customers' waiting incentive, that is, $q_1^{\text{S-DR}}(\theta) \leq q_1^{\text{M-DR}}(\theta)$ as shown in the proof of Proposition 3.4.2. This on average leads to more unsatisfied high-end customers from period 1. Knowing that there are more customers who can buy V2, the firm has more incentive to innovate. Second, when selling to strategic customers, a higher innovation in V2 not only increases the price for V2, but also implies a higher chargeable price for V1 in period 1 as the indirect positive behavioral impact shown in (3.7). There is a chain-effect here: the innovation level of V2 affects the inventory level of V1, which in turn affects the price of V1. The key part is the interdependence between the inventory level and the price of V1, which is absent when selling to myopic customers. Because of this interdependence, *a higher innovation level can mitigate the waiting behavior.*

3.4.2 Single Rollover (S-SR)

The analysis in period 2 is the same as that in Section 3.3.2. If $q_1 < N$, then there is no V1 left and the firm sets $p_2 = v(1+\theta)$, which implies a zero surplus from waiting. If $q_1 > N$, then there are leftover V1 and the firm sell V2 at $p_2 = v(1+\theta)$ to newcomers. However, different from dual rollover, when $q_1 > N$, the firm under single rollover will dispose of the leftover V1 outside of the market, which implies again a zero surplus from waiting. Therefore, under single rollover, the waiting surplus is zero for strategic customers. This means that strategic customers buy V1 in period 1 when $p_1 = v$, the same price charged to myopic customers. Here, we reach the result comparing the firm's profit and innovation investment under single rollover when selling to different customers.

Proposition 3.4.3 (M-SR vs. S-SR) *Under single rollover, strategic customers behave myopically. In addition, under single rollover, the firm earns the same profit and innovates at the same level regardless of whether customers are myopic or strategic. That is,* $\Pi^{\text{M-SR}} = \Pi^{\text{S-SR}}$ *and* $\theta^{\text{M-SR}} = \theta^{\text{S-SR}}$*.*

Therefore, by adopting single rollover, the firm can completely eliminate strategic waiting. However, eliminating strategic waiting does not guarantee a higher profit. This is because the firm suffers from the low disposal value of the leftover V1 under single rollover as implied in Lemma 3.4.4.

Lemma 3.4.4 (S-SR) *The firm's optimal total expected profit* $\Pi^{\text{S-SR}}$ *increases in* σ *while the optimal innovation level* $\theta^{\text{S-SR}}$ *decreases in* σ*.*

Proof. We can prove Lemma 3.4.4 in a similar way to that in Proposition 3.3.2 by considering $\delta = \sigma_1$, $\sigma = \sigma_2$, and $\sigma_1 > \sigma_2$. ■

Proposition 3.4.5 i) below shows that single rollover can outperform dual rollover only when the market-disposal spread $(\delta - \sigma)$ is not large.

Proposition 3.4.5 (S-SR vs. S-DR) *i) There exists a threshold* $\triangle_\Pi \geq 0$ *such that when the market-disposal spread* $\delta - \sigma \leq \triangle_\Pi$*, then* $\Pi^{\text{S-SR}} \geq \Pi^{\text{S-DR}}$*; otherwise,* $\Pi^{\text{S-SR}} < \Pi^{\text{S-DR}}$*.*
ii) There exists a threshold $\triangle_\theta \geq 0$ *such that when the market-disposal spread* $\delta - \sigma \leq \triangle_\theta$*, then* $\theta^{\text{S-SR}} \leq \theta^{\text{S-DR}}$*; otherwise,* $\theta^{\text{S-SR}} > \theta^{\text{S-DR}}$*.*

Proof. When $\sigma = \delta$, we have $\Pi^{\text{S-SR}} = \Pi^{\text{M-SR}} = \Pi^{\text{M-DR}} \geq \Pi^{\text{S-DR}}$, where the first equality is from Proposition 3.4.3, the second equality is from the fact that with myopic customers, the only difference between single and dual rollover lies in σ vs. δ, and the last inequality is from Proposition 3.4.2. Because $\Pi^{\text{S-SR}}$ increases in σ according to Lemma 3.4.4, if we decrease σ by starting with $\sigma = \delta$, then there exists a threshold $\triangle_\Pi \geq 0$ such that when $\delta - \sigma \leq \triangle_\Pi$, we have $\Pi^{\text{S-SR}} \geq \Pi^{\text{S-DR}}$; otherwise, $\Pi^{\text{S-SR}} < \Pi^{\text{S-DR}}$. This completes the proof of i).

Similarly, when $\sigma = \delta$, then $\theta^{\text{S-SR}} = \theta^{\text{M-SR}} = \theta^{\text{M-DR}} \leq \theta^{\text{S-DR}}$. Because $\theta^{\text{S-SR}}$ decreases in σ according to Lemma 3.4.4, if we decrease σ by starting with $\sigma = \delta$, then there exists a threshold $\triangle_\theta \geq 0$ such that when $\delta - \sigma \leq \triangle_\theta$, we have $\theta^{\text{S-SR}} \leq \theta^{\text{S-DR}}$; otherwise, $\theta^{\text{S-SR}} > \theta^{\text{S-DR}}$. ■

Comparing Propositions 3.3.2 and 3.4.5, when customers are myopic, the firm *always* gets a lower profit but invests more in innovation under single rollover, when compared to dual rollover. However, this is not true with strategic customers, in which case, depending on the value of the market-disposal spread, the firm can earn either a higher or a lower profit and invest either more or less in innovation under single rollover. This is because when customers are myopic, compared to dual rollover, a low disposal value under single rollover hurts the profit but helps the innovation level. When customers are strategic, however, besides the disposal value, another element – the waiting behavior – influences the results. The waiting behavior lowers the profit since the firm cannot extract all of the customers' utility, and thus has to reduce the price to induce early purchasing, but the waiting behavior speeds up the innovation process; see our discussion following Proposition 3.4.2.

Therefore, regarding the profit, a firm adopting single rollover suffers from the low disposal value of V1, but benefits from the eliminated waiting behavior. When the disposal value of V1 under single rollover is not relatively low (i.e., $\delta - \sigma \leq \triangle_\Pi$), the waiting behavior effect on the firm's profit dominates the low disposal value effect, which leads to a higher profit than that under dual rollover (i.e., $\Pi^{\text{S-SR}} \geq \Pi^{\text{S-DR}}$). In terms of the innovation level, under single rollover, the low disposal value of V1 forces the firm to innovate faster, but the eliminated waiting incentive retards the innovation. When the disposal value of V1 under single rollover is not low enough (i.e., $\delta - \sigma \leq \triangle_\theta$), the waiting behavior effect dominates and thus leads to a lower innovation level (i.e., $\theta^{\text{S-SR}} \leq \theta^{\text{S-DR}}$).

According to Proposition 3.3.2, when selling to myopic customers, it is *not* possible for the firm to invest more in innovation and at the same time earn a higher profit, regardless of the rollover strategy adopted. A natural question that arises is whether the same is possible when customers are strategic. The answer is yes, as shown in Proposition 3.4.6.

Proposition 3.4.6 *i) If* $\triangle_\Pi > \triangle_\theta$*, then* $\Pi^{\text{S-SR}} > \Pi^{\text{S-DR}}$ *and* $\theta^{\text{S-SR}} > \theta^{\text{S-DR}}$ *for* $\triangle_\theta < \delta - \sigma < \triangle_\Pi$.
ii) If $\triangle_\Pi < \triangle_\theta$*, then* $\Pi^{\text{S-DR}} > \Pi^{\text{S-SR}}$ *and* $\theta^{\text{S-DR}} > \theta^{\text{S-DR}}$ *for* $\triangle_\Pi < \delta - \sigma < \triangle_\theta$.

Proof. The proof uses the results of Proposition 3.4.5 in two separate cases: $\triangle_\Pi > \triangle_\theta$ and $\triangle_\Pi < \triangle_\theta$. ■

Proposition 3.4.6 states that when selling to strategic customers, with a moderate market-disposal spread ($\min\{\triangle_\Pi, \triangle_\theta\} < \delta - \sigma < \max\{\triangle_\Pi, \triangle_\theta\}$), the firm provides a more innovative product in order to earn a higher profit under the appropriate rollover strategy (either single rollover or dual rollover, depending on the relation between $\triangle_\Pi$ and $\triangle_\theta$) compared to the other rollover strategy. It underscores the importance of adopting the appropriate rollover strategy when selling to strategic customers.

3.5 Conclusion

We investigate how different rollover strategies and strategic waiting behavior impact a firm's innovation level and profit in four settings. We show that when customers are myopic, single rollover hurts the firm's profit, but *surprisingly*, increases the innovation level. The presence of strategic customers reduces the firm's profit but forces the firm to innovate faster, which is *counterintuitive*. We also show that when customers are strategic, the innovation level and the profit can both be improved by selecting a proper rollover strategy, whereas this is not the case when customers are myopic. In addition, when selling to strategic customers, the firm can use single rollover to eliminate strategic customers' waiting behavior completely while extracting all of their utility.

We study a two-period model in order to get analytical results and clear insights, and thus do not consider the option of delaying the introduction of the new product. Delaying introduction of the new product can mitigate strategic waiting behavior by increasing customers' future utility discount and increasing the stockout risk of the current product. Thus, with this option, a firm has more incentives to adopt dual rollover rather than single rollover. However, postponing product introduction can hurt the firm when the future utility discount is too high. Because a firm can compensate the future discount by providing a more innovative new product, it may need to consider the tradeoff between the timing of product introduction and the cost of R&D. For detailed studies of product introduction and phase-out timing, see Lim and Tang (2006), Arslan et al. (2009), and Koca et al. (2010).

CHAPTER 4

SITE-TO-STORE OR STORE-TO-SITE? APPLICATION OF ONE-WAY TRANSSHIPMENT IN DUAL-CHANNEL RETAILING

4.1 Synopsis

For many retailers, "the question of whether or not to adopt a multi-channel retailing strategy has already been answered: multi-channel retailing is a business imperative." (Reportlinker 2010). In a few short years, the dramatic development of shopping through the Internet has seen many traditional "brick-and-mortar" retailers, such as Walmart, J.C. Penney, Target and Macy's launching new virtual online channels, and thus becoming "brick-and-click" retailers. Over time, some pure internet retailers such as iParty.com have expanded their businesses by opening physical stores (Agatz et al. 2008). As a result, multi-channel presence is now a dominant strategy for retailers, and learning how to integrate channels effectively is key to their success. A report for top UK e-retail strategies ranks "multi-channel integration" as the top priority for e-retailing success (IMRG 2006).

With orders from multiple channels, "Every retailer will need to have the capability of taking orders from anywhere and fulfilling from anywhere, if not now, then at some point down the road." (Internet Retailer 2009). In order to achieve the goal of "Buy Anywhere, Fulfill Anywhere", sharing inventory across channels is important. In fact, without inventory sharing across channels, most of the cross-channel activities cannot be done (Bengier 2010). Many retailers who initially separated inventory for their physical and online stores, are now looking at pooled inventory. "It [Pooled inventory] allows greater inventory control, cross-channel ordering...."(Faithful 2010). "When retailers combine cross-channel order management with the ability to fulfill orders from any channel, including stores and warehouses dedicated for either stores or web sales, they can better match customer demand with available inventory." (Internet Retailer 2009). For example, endless aisles, one of the key strategies leading to a successful multi-channel retailing, can effectively capture lost sales caused by the unavailable items in the store by enabling customers to access the inventory across the whole retail chain (Kramer 2008).

However, channel integration is not easy due to the different demand drivers, optimal inventory configurations, cost structures, product varieties, delivery mechanisms, and so on (Metters and Walton 2007). One of the three key reasons why some retailers shun the multi-channel strategy is the operational difficulty of integration (Zhang et al. 2010). A study, based on a survey of 225 companies in the consumer product distribution industry, argues that the biggest operational challenge facing multi-channel retailers is lack of integration between inventory and order management systems (Business Wire 2004). Therefore, it is important to study how a multi-channel retailer can effectively integrate different channels.

From a supply chain's perspective, integrating traditional and online channels is attractive due to cost savings in holding inventory, improved customer service resulting from reduction in lost sales, risk pooling through inventory sharing, and potential economies of scale. In practice, different strategies have been pursued by different organizations to integrate the two channels.

One channel integration strategy is the *store-to-site* strategy. With this strategy, a retailer integrates the two channels by transparently making in-store inventory available to online orders when the online warehouse is out of stock. According to Rupp (2009), this strategy is adopted by such retailers as Orvis Company Inc., Systemax Inc.'s CompUSA, and Jones Apparel Group (the holding company for popular apparel and footwear brands including Jones New York and Nine West). Some retailers such as Tesco, Gap, and Fnac (the largest retailer of books, music, and software in France) follow a variant of the store-to-site strategy to integrate the two channels. These firms use inventories in their existing retail stores rather than build warehouses to fulfill their online demands (Seifert et al. 2006).

Another channel integration strategy is the *site-to-store* strategy. With this strategy, a retailer keeps a central warehouse for his online demand. In addition, a customer, who is not able to find the desired product in a local retail store, has the choice to order the product from the retailer's web site through an in-store online shopping system, and then pick it up from the local store or have it delivered directly to home. The in-store online shopping system can be a kiosk or even a salesperson's handheld device (Kramer 2008). For example, both Adidas and Walmart have kiosks or computer terminals installed in some of their retail stores, through which customers can order out-of-stock items. The department store chain Kohl's plans to deploy self-service kiosks in all of its stores in late 2010 after a successful pilot test in 2009, and expects these kiosks to be another major driver of its market share (Gokis 2010). Dell uses a variant of the site-to-store strategy for multi-channel retailing. Instead of stocking inventory in retail stores, Dell places sample products for computers and printers in selected retail stores so that customers can experience the products and then order them through in-store kiosks (WSJ 2003).

Besides the site-to-store and store-to-site strategies, other approaches exist to integrate the two channels. According to a survey of 55 dual-channel retailers, when online demand is very small, some retailers use existing distribution centers, which supply physical stores, to fulfill online orders (de Koster 2003). *Unfortunately, different firms seem to be experimenting with different integrating approaches without knowing which method works best for them* (Bendoly et al. 2007).

In this chapter, we take a first step to study two standard integration strategies: site-to-store and store-to-site. In particular, we answer the following questions: (i) How should a retailer integrate his two channels – site-to-store or store-to-site? (ii) When does a channel integration strategy matter? To answer the two questions, we model a dual-channel retailing system as a newsvendor network (Van Mieghem and Rudi 2002). The newsvendor network consists of a channel with a single store and another channel with multiple stores. In our analysis, we shall refer to the former as an online channel and the latter a physical channel, rather than call them channel one and channel two. The terminology notwithstanding, our analysis applies to retail systems with two independent channels.

We first consider the case in which the physical channel consists of one retail store. We demonstrate that if inventory can be stored in only one channel, then it should be stored in the channel with stochastically larger or less uncertain demand (in the concave order). We then study the channel integration strategy for the case in which inventory can be kept in both channels by assuming demands to follow three-point distributions. It turns out that the optimal channel integration strategy ("the optimal strategy" for short, hereafter) depends on the product contribution margin and the channel demand distribution shape. In particular, for high-margin products, the firm should use the excess inventory from the channel with low (resp., high) demand variability to satisfy the extra demand in the channel with high (resp., low) demand variability when the probability for the middle value is high (resp., low). For low-margin products, the transshipment direction should be the opposite.

We then use numerical experiments to study the channel integration problem for normal and gamma demand distributions. Our experiments show that the insights developed with three-point demand distributions continue to hold for normal and gamma distributions. Furthermore, the optimal strategy makes a significant difference on the firm's profit when the margin is very high or very low, when the transshipment cost is high, when the demand correlation between the two channels is not strongly positive, or when the difference between the demand uncertainties of the two channels is large.

Next, we study the case in which there are multiple retail stores in the physical channel. Based on our results with three-point distributions, we develop a simple rule to identify an effective integration strategy for the multiple-retail-store case with normal demands. With this rule, one only needs to compare the demand standard deviation of the online channel with the sum of the demand standard deviations of all the retail stores. Our computational experiments show that this simple heuristic identifies the optimal strategy in 169 out of 180 problem instances. Moreover, we find that the number of retail stores affects the integration strategy significantly. In particular, when the number of retail stores is large, a firm should choose a site-to-store (resp., store-to-site) strategy when the product contribution margin is high (resp., low).

Finally, we build a circular spatial model for dual-channel retailing systems. The circular model captures the main factors that drive customer purchasing behavior, such as the travelling distance to a retail store and the inconvenience of shopping online. We demonstrate how our results can be applied to this circular model and develop insights on how customer purchasing behavior drivers impact a retailer's channel integration strategy.

To characterize the optimal strategy, we need to compare the optimal solutions of two one-way transshipment problems, which turns out to be quite a challenge for demands with general distributions. One might think that the novel approach used by Lu and Van Mieghem (2009) may be applied to our problem. However, in their model, transshipment is used in anticipation of future mismatch between supply and demand. Consequently, it is possible that one-way or no transshipment can be the optimal strategy in Lu and Van Mieghem (2009) under certain conditions. On the other hand, in our model, transshipment is used as a recourse action after the mismatch happens. Therefore, transshipment always benefits the firm whenever the mismatch occurs as long as the transshipment cost is reasonably small. As a result, the approach of Lu and Van Mieghem (2009) cannot be applied directly to our problem. Therefore, we assume that demands follow three-point distributions to develop analytical results.

The three-point distribution for demand has not been assumed in the supply chain literature; however, two-point distributions, the special cases of three-point distributions, have been widely considered (Desai et al. 2007, Padmanabhan and Png 1997, Yang and Schrage 2009, Deneckere et al. 1997, Andersson et al. 1998, Desai et al. 2001 and Kumar et al. 2001). While a two-point distribution assumption makes analysis tractable, it has the disadvantage in that a two-point distribution cannot be used to properly approximate unimodal distributions (such as normal distribution) that are commonly observed in practice and analyzed in the literature. Therefore, we assume that channel demands follow three-point distributions. Three-point distributions can be used to effectively approximate many other continuous distributions such as normal, lognormal and beta distributions (Keefer and Bodily 1983, Keefer 1994). In reality, this approximation has already been widely applied in decision and risk analyses such as decision trees (Cobb 2011) and asset valuation (Tseng and Lin 2007, Nosic and Weber 2010), and in engineering-economic analyses such as drug design in pharmaceutical industry (S.Stonebraker and L.Keefer 2009, Stonebraker 2002) and R&D projects (Perdue et al. 1999). It turns out that the assumption of three-point distributions is critical to our results: The channel integration strategy may be different for unimodal demands and bimodal demands.

This chapter is organized as follows. Section 4.2 reviews the literature. Section 4.3 specifies the assumptions and develops the model. We consider the case with only one retail store in the physical channel in Section 4.4, and conduct numerical experiments in Section 4.5 to test the robustness of our analytical results as well as investigate the multiple-retail-store case. Section 4.6 presents a circular model for dual-channel retailing systems, and Section 4.7 concludes the chapter.

4.2 Literature Review

The integrated dual-channel retailing system is modeled as a newsvendor network (Van Mieghem and Rudi 2002), where multiple resources are utilized to meet different streams of demand. In our model, resources are re-allocated to meet a different stream of demand via transshipment. Therefore, our work is related to the vast literature on transshipment. The existing work on transshipment focuses on the optimal inventory ordering and allocation policies in centralized systems (see, for example, Krishnan and Rao 1965, Tagaras 1989, Robinson 1990, Tagaras and Cohen 1992, Bassok et al. 1999, Herer and Rashit 1999, Axsäter 2003, Wee and Dada 2005, Herer et al. 2006, and Çömez et al. 2011b) or the coordination issues existing in decentralized systems (Rudi et al. 2001, Anupindi et al. 2001, Granot and Sošić 2003, Sošić 2006, and Çömez et al. 2011a).

Our work departs from these two streams of research in that we focus on the network design problem with transshipment. In their seminal work, Jordan and Graves (1995) demonstrate that limited flexibility yields most of the benefits of total flexibility and a chaining strategy is effective in providing limited flexibility. Our work is similar in spirit to Jordan and Graves (1995) in that we study how to incorporate limited flexibility in a newsvendor network. However, our focus is on the direction of the added flexibility instead of its degree. Lu and Van Mieghem (2009) and Dong et al. (2010) examine the issue of flexibility direction in the context of global facility network design using a newsvendor network. Lu and Van Mieghem (2009) demonstrate that the direction of flexibility depends on the demand characteristics and relative magnitude of price and manufacturing cost differentials. Dong et al. (2010) study how exchange rate uncertainty and responsive pricing affect the optimal network configuration. In contrast to these two papers, we assume that there are no cost and price differences between the two channels to distill the impact of the channel demand characteristics on the channel integration strategy. In addition, we also study the impact of the number of retail stores, which has not been addressed in the earlier studies.

Seifert et al. (2006) study the channel integration problem of a multi-channel retailing system from a supply chain's perspective and demonstrate that the cost savings from an integrated system using the store-to-site strategy can be significant. However, they compare the performance of the store-to-site strategy to the performance without any integration (two independent channels), rather than compare the performance of different integration strategies as we do in this chapter. Alptekinoglu and Tang (2005) and Bendoly et al. (2007) also compare dual-channel retailing systems with the "store-to-site" strategy to those without integration. They do not allow transshipments between the two channels as a recourse action. Instead, they allocate a fraction of online orders to be fulfilled by the physical channel in anticipation of future demand, and the unsatisfied demand of either channel is backordered.

Online retailing has brought other new challenges to both practitioners and researchers. Bhargava et al. (2006) and Sun et al. (2008) study stockout compensation and stockless operation phenomenon that are commonly practiced in the online retailing industry. However, they do not consider the channel integration strategy under a dual-channel environment. Online retailing also changes the interactions between different members of a supply chain. Within the dual-channel retailing literature, many recent papers focus on the strategic interaction between a manufacturer who operates an online channel and a retailer who operates retail stores (Chiang et al. 2003, Tsay and Agrawal 2004, Cattani et al. 2006, Ryan et al. 2008, and Chen et al. 2008). Differing from this stream of research, we focus on the integration of the physical and online channels that are operated by a single firm.

4.3 Model and Assumptions

We consider the channel integration problem of a retailer who sells one product in a dual-channel retailing system. The dual-channel system consists of an online store and n retail (physical) stores. The retailer can integrate the two channels using two alternative strategies: site-to-store and store-to-site. In both strategies, each channel fulfills the customer demand from its own inventory first. With the site-to-store strategy, the excess inventory in the online channel is used to fill the unmet demands of the retail stores. With the store-to-site strategy, the excess inventory in the retail stores is used to fill the unmet demand of the online channel. The retailer chooses the strategy that maximizes his expected profit.

We study the retailer's channel integration decisions by modeling the dual-channel retailing system as a newsvendor network. We focus on a single-period model for convenience in exposition and for gaining clear insights. Our model assumptions satisfy the conditions defined in Ignall and Veinott (1969). Therefore, a multi-period inventory optimization problem can be reduced to a newsvendor network problem (see, also, Nahmias and Smith (1994), McGavin et al. (1997), and Agrawal and Smith (2000) for discussion). Consequently, our analysis and insights can be extended to multi-period cases.

At the beginning of the period, the retailer decides, for the online store and each retail store, respectively, the nonnegative order quantities Q_o and Q_{ri}, before observing demands D_o and D_{ri}. Here, the subscript o and ri refer to online store and retail store i, respectively, where $i = 1, ..., n$. When there is only one retail store, we drop the subscript i, and use Q_r and D_r to denote the retail store's order quantity and demand, respectively. After the demands materialize in both channels, each channel first fills its own demand. Then, the leftover inventory from one channel is used to fill the excess demand of the other channel based on the integration strategy.

We assume that the unit ordering cost is c and the unit (retail) price is p in both channels; see Cattani et al. (2006) for an argument in favor of the identical price assumption in the two channels. In addition, when the site-to-store strategy is adopted, each unit of inventory from the online store warehouse to fulfill the demand in a retail store incurs a transshipment cost τ_{OR} to the retailer. This transshipment cost can represent the transportation cost from the online store warehouse to the retail store or to the customer's home. It can also be considered as the order-fulfillment handling cost in the online store warehouse. When the store-to-site strategy is adopted, the shipping cost for home delivery is usually borne by the customers. However, because the retail stores are not designed for individual-product handling, the online order fulfillment in the retail stores is not as efficient as that in the online store, which leads to a relatively high handling cost. Let τ_{RO} be the cost of handling in excess of what is incurred in the online store. We assume that $\tau_{\mathrm{OR}} = \tau_{\mathrm{RO}} = \tau$ to isolate the effects of product contribution margin and demand characteristics on the integration strategy. For simplicity, we assume no salvage value for the leftover inventory and no penalty cost for lost sales, although they can be incorporated into our model with additional notations, but without additional insights. To avoid trivial cases, we assume $p > c$ (for the retailer to participate) and $p > \tau$ (for transshipment to be beneficial). Let $F_x(d) = \Pr[D_x \leq d]$ denote the cumulative distribution function (CDF) of the random variable D_x. We call the retailing system adopting the store-to-site strategy (resp., the site-to-store strategy), the RO system (resp., the OR system), where R represents Retail store and O represents Online store.

4.4 Integration Strategy with One Retail Store

In this section, we assume that there is only one retail store in the physical channel, and we study two scenarios. In the first scenario (Section 4.4.1), inventory is allowed in one channel only – either in the physical channel (retail store) or in the online channel (online store). This scenario corresponds to the industry practice followed by Tesco, Gap, and Fnac for the store-to-site strategy and by Dell for the site-to-store strategy. In the second scenario (4.4.2), we examine the integration strategy when inventory can be kept in both channels. The second scenario corresponds to the industry practice followed by Orvis Company Inc., Systemax Inc.'s CompUSA, and Jones Apparel Group for the store-to-site strategy and by Adidas, Walmart, and Kohl's for the site-to-store strategy. Whether the inventory is stocked in only one channel or in both is assumed to be given, and so we focus on the optimal strategy under the two scenarios: stocking inventory in one channel only or in both channels.

4.4.1 Inventory in One Channel Only

First, we consider the RO system, and the OR system can be assessed in a similar manner. In the RO system, the retailer orders Q_r for the retail store and uses it to meet demands in both channels. The retailer's expected profit $\pi^{\mathrm{RO}}(Q_r)$ can be written as

$$\pi^{\mathrm{RO}}(Q_r) = -cQ_r + E[p\min(Q_r, D_r) + (p-\tau)\min((Q_r - D_r)^+, D_o)]. \tag{4.1}$$

This expression consists of the ordering cost cQ_r, the expected revenue $E[p\min(Q_r, D_r)]$ from the sales in the retail store, and the expected revenue (net of transshipment cost) $E[(p-\tau)\min((Q_r - D_r)^+, D_o)]$ from the online store. It is easy to prove that the retailer's optimal order quantity is determined by

$$\frac{\tau}{p}F_r(Q_r) + \frac{p-\tau}{p}F_+(Q_r) = \frac{p-c}{p}, \tag{4.2}$$

where $F_+(\cdot)$ denotes the distribution function of $D_+ = D_r + D_o$. An important observation is that the left-hand side of (4.2) is a distribution function, because it is a mixture of two distribution functions, and the right-hand side is the newsvendor critical ratio. Therefore, the RO system is equivalent to a newsvendor problem with the purchasing cost c, the retail price p, and the demand

$$D_{\text{RO}} = \begin{cases} D_r \text{ with probability of } \frac{\tau}{p}, \\ D_+ \text{ with probability of } \frac{p-\tau}{p}. \end{cases} \tag{4.3}$$

Similarly, the OR system is equivalent to a newsvendor problem with the purchasing cost c, the retail price p, and the demand

$$D_{\text{OR}} = \begin{cases} D_o \text{ with probability of } \frac{\tau}{p}, \\ D_+ \text{ with probability of } \frac{p-\tau}{p}. \end{cases} \tag{4.4}$$

Here we should point out that Zhang (2005) established the equivalence between an inventory problem with transshipment and a newsvendor problem, and used the equivalence result to extend the anaysis of Dong and Rudi (2004) to general demand distributions. Our equivalence result is similar to that in Zhang (2005); however, the transshipment in our model is one way while it is two way in Zhang (2005).

By comparing (4.3) and (4.4), we notice that the RO and OR systems face the same demand D_+ with probability $\frac{p-\tau}{p}$, whereas the RO system faces the demand D_r with probability $\frac{\tau}{p}$ and the OR system faces the demand D_o with the same probability. To compare the performance of the two systems, we compare their demands in two stochastic orders: the usual stochastic order and concave order. A random variable X is larger in the usual stochastic order (resp., in the concave order) than a random variable Y if and only if for all increasing (resp., concave) functions f we have $E[f(X)] \geq E[f(Y)]$ (Müller and Stoyan 2002). In this chapter, we term "non-decreasing" (resp., "non-increasing") simply as "increasing" (resp., "decreasing"). Since the profit of a newsvendor is increasing and concave in the demand realization, the expected profit is increasing when the random demand increases in the usual stochastic or concave order. Let π^{RO} and π^{OR} be the respective optimal expected profits of the RO and OR systems. Proposition 4.4.1 characterizes the optimal strategy when inventory is to be kept only in one channel.

Proposition 4.4.1 *(i) If $D_r \geq_{st} D_o$ (resp., $D_r \leq_{st} D_o$), that is, D_r is larger than (resp., smaller than) D_o in the usual stochastic order, then $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$ (resp., $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$). (ii) If $D_r \geq_{cv} D_o$ (resp., $D_r \leq_{cv} D_o$), that is, D_r is larger than (resp., smaller than) D_o in the concave order, then $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$ (resp., $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$).*

Proposition 4.4.1 states that if the retailer can choose only one channel to stock inventory, he should choose the channel with (stochastically) larger demand or the channel with less uncertain demand (in the concave order). It is important to point out that the result does not necessarily hold if we only compare the average demands of the two channels, that is, if $\mu_r \geq \mu_o$ (resp., $\mu_r \leq \mu_o$), we cannot conclude that $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$ (resp., $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$).

4.4.2 Inventory in Both Channels

In this section we allow both channels to keep inventory. Recall that the order quantities of the retail and online stores in the RO system are denoted as Q_r and Q_o, respectively. The expected profit of the RO system can be written as

$$\begin{aligned}\pi^{\mathrm{RO}}(Q_r, Q_o) &= -c(Q_r + Q_o) + E[p\min(Q_r, D_r) \\ &\quad +p\min(Q_o, D_o) + (p-\tau)\min\{(Q_r - D_r)^+, (D_o - Q_o)^+\}]. \qquad (4.5)\end{aligned}$$

Similar, the retailer's expected profit of the OR system is

$$\begin{aligned}\pi^{\mathrm{OR}}(Q_r, Q_o) &= -c(Q_r + Q_o) + E[p\min(Q_r, D_r) \\ &\quad +p\min(Q_o, D_o) + (p-\tau)\min\{(Q_o - D_o)^+, (D_r - Q_r)^+\}]. \qquad (4.6)\end{aligned}$$

Owing to the difficulty in obtaining analytical results for demands with general distributions, we consider the situation in which demands D_r and D_o follow three-point distributions. Specially, D_r takes three values $\mu_r - \delta_r$, μ_r, and $\mu_r + \delta_r$ with probability k, $1-2k$, and k, respectively. Similarly, D_o takes three values $\mu_o - \delta_o$, μ_o, and $\mu_o + \delta_o$ with probability

k, $1-2k$, and k, respectively. Thus, $E[D_r] = \mu_r$, $Stdev[D_r] = \sqrt{2k}\delta_r$, $E[D_o] = \mu_o$ and $Stdev[D_o] = \sqrt{2k}\delta_o$. We further assume that $0 \leq k \leq 1/2$ to ensure positive probabilities for all three values. Lemma 4.4.2 demonstrates that the optimal strategy is independent of demand means.

Lemma 4.4.2 $\pi^{\text{RO}} - \pi^{\text{OR}}$ *is independent of* μ_r *or* μ_o*.*

Lemma 4.4.2 is intuitive, because the expected profits of both systems are affected by the demand means only through their total (that is, $\mu_r + \mu_o$). This implies that any change in a demand mean will change the expected profits of both systems equally, and therefore it will not impact the relation between the two systems. Therefore, without loss of generality, we assume $\mu_r = \mu_o = \mu$ in the remainder of this section, since all our results with this assumption continue to hold when $\mu_r \neq \mu_o$. Next, we examine how the second moment of demand affects the channel integration strategy. Since when $\delta_r = \delta_o$, OR and RO systems are equivalent, we are interested in developing insights when δ_o and δ_r are different. Without loss of generality, we assume that $\delta_o > \delta_r$. With this assumption, we have $Stdev[D_o] > Stdev[D_r]$. To develop insights that are transparent and easy to communicate, we further assume $\delta_o/\delta_r \geq 2 - \tau/c$. Our computational studies indicate that the key insights continue to hold when this condition is violated. More importantly, the channel integration strategy will not affect the firm's profit significantly when this condition is violated. To present our main result, we introduce some notations. Let

$$k_1 := [(p-c)(\delta_o - 2\delta_r) + \tau\delta_r]/[p\delta_o - 2(p-\tau)\delta_r],$$

and $k_2 \in [0, 1/2]$ be the unique root of

$$(p-c)(\delta_o - \delta_r) + [p\delta_r k - p\delta_o k + (p-\tau)\delta_r k^2] = 0$$

if the root exists; otherwise we set k_2 to $1/2$. The values of $k_4, \cdots, k_8$ below are set similarly.

Let

$$k_3 := [c(\delta_o - 2\delta_r) + \tau\delta_r]/[p\delta_o - 2(p-\tau)\delta_r],$$

and $k_4, k_5, k_{6,}k_7, k_8 \in [0, 1/2]$ be the unique roots, respectively, of the following equations

$$c(\delta_o - \delta_r) + pk(\delta_r - \delta_o) + (p-\tau)k^2\delta_r = 0) = 0,$$

$$(2c-p)\delta_r + (p-c-pk)\delta_o + (p-\tau)[\delta_r(1-2k)(2k-1) - 2\delta_r k^2 + \delta_o k^2 - k(1-2k)\delta_o] = 0,$$

$$p - c - pk + (p-\tau)k^2 = 0,$$

$$(p-2c)\delta_r + (c-pk)\delta_o + (p-\tau)[\delta_r(1-2k)(2k-1) - 2\delta_r k^2 + \delta_o k^2 - k(1-2k)\delta_o] = 0,$$

and

$$pk - c - pk^2 + \tau k^2 = 0.$$

Define

$$\bar{k} := \begin{cases} \min\{k_1, k_2\} & \text{if } \delta_o/\delta_r \geq 2 \text{ and } p \leq c/(1-k), \\ 1/2 & \text{if } \delta_o/\delta_r \geq 2 \text{ and } c/(1-k) < p \leq 2c, \\ \min\{k_5, k_6\} & \text{if } 2 - \tau/c \leq \delta_o/\delta_r < 2 \text{ and } p \leq c/(1-k), \\ k_5 & \text{if } 2 - \tau/c \leq \delta_o/\delta_r < 2 \text{ and } c/(1-k) < p \leq 2c, \end{cases}$$

$$\hat{k} := \begin{cases} 1/2 & \text{if } \delta_o/\delta_r \geq 2 \text{ and } 2c < p < c/k, \\ \min\{k_3, k_4\} & \text{if } \delta_o/\delta_r \geq 2 \text{ and } p \geq c/k, \\ k_7 & \text{if } 2 - \tau/c \leq \delta_o/\delta_r < 2 \text{ and } 2c < p < c/k, \\ \min\{k_7, k_8\} & \text{if } 2 - \tau/c \leq \delta_o/\delta_r < 2 \text{ and } p \geq c/k. \end{cases}$$

Now, we can present the main result of this chapter.

Proposition 4.4.3 *i)* $p = 2c$, $\pi^{\mathrm{RO}} = \pi^{\mathrm{OR}}$.
ii) If $p < 2c$, *when* $k \leq \bar{k}$, $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$, *otherwise* $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$.
iii) If $p > 2c$, *when* $k \leq \hat{k}$, $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$, *otherwise* $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$.

From Proposition 4.4.3, the optimal strategy depends on the contribution margin of the product and the demand distribution shape (in terms of k). When $p = 2c$, the site-to-store and store-to-site systems are equivalent. For low-margin products (when $p < 2c$), the site-to-store system (weakly) dominates the store-to-site system when k is small; the store-to-site system (weakly) dominates the site-to-store system when k is large. For high-margin products (when $p > 2c$), the dominance relationships between the two systems are the other way around.

To better understand the impacts of product contribution margin and channel demand distribution shape on the firm's optimal strategy, we plot the firm's optimal strategy in the (k, p)-plane in Figure 4.1. When k is small, the two integration strategies are equivalent. When k is large, however, the optimal transshipment directions for low-margin and high-margin products are opposite to one another. Specifically, when $p = 2c$, the two transshipment directions yield the same profit for the firm regardless of the value of k. For high-margin products, as k increases, the firm's optimal strategy will switch from store-to-site to site-to-store. However, for low-margin products, as k increases, the firm's optimal strategy switches from site-to-store to store-to-site. The following corollary regarding the optimal strategy for two-point demand distributions follows directly from Proposition 4.4.3.

Corollary 4.4.4 *Assume $k = 0.5$. Then, $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$ if $p < 2c$; $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$ if $p > 2c$.*

The fact that the two channel integration strategies yield the same profit for the firm when k is small is not surprising. Intuitively, the firm's demand uncertainty is low for a small k value. As a result, transshipment does not make a significant impact on the firm's profit. Consequently, the firm is indifferent to the two integration strategies. In the extreme case of deterministic demand (when $k = 0$), the firm never needs to transship from one channel to the other.

The impacts of margin and demand distribution shape (in terms of k) on the optimal strategy are driven by the following three facts. First, following a standard newsvendor problem argument, for high-margin products, the total optimal order quantities tend to be greater than the total demand mean; for low-margin products, the total order quantity tend

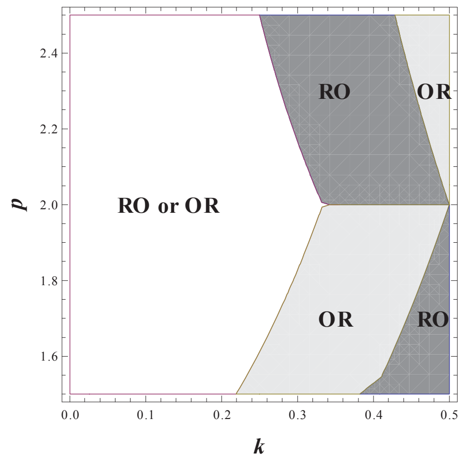


Figure 4.1. Optimal Channel Integration Strategies ($c = 1,\ \tau = 0.5,\ \delta_o = 3,\ \delta_r = 1$)

to be less than the total demand mean. Second, in general, the optimal order quantity for the sourcing channel tends to be greater than the demand mean, and the optimal order quantity for the destination channel tends to be less than the demand mean. Third, the proper channel integration strategy should make a tradeoff between the local sales and the transshipment quantity.

Consider high-margin products (the case for low-margin products follow similarly). Suppose that k is relatively small. In this case, the probability that demand takes the middle value is relatively high. If the firm follows the OR strategy, we would expect the optimal order quantity to be $(Q_r, Q_o) =(\mu, \mu + \delta_o)$. On the other hand, if the firm follows the RO strategy, the optimal order quantity would be $(Q_r, Q_o) = (\mu + \delta_r, \mu)$. In both strategies, we expect the total local sales to be close to 2μ because k is small. However, the transshipment quantity in the OR strategy is $\min(\delta_r, 2\delta_o)$ with probability k^2 and is $\min(\delta_r, \delta_o) = \delta_r$ with probability $(1 - 2k)k$. Similarly, the transshipment quantity in the RO strategy is $\min(2\delta_r, \delta_o)$ with probability k^2 and $\min(\delta_r, \delta_o) = \delta_r$ with probability $(1 - 2k)k$. Clearly, $\min(2\delta_r, \delta_o) > \min(\delta_r, 2\delta_o)$. Consequently, the firm would prefer the RO strategy since it makes better use of the transshipment option.

Now suppose that k is relatively large, which implies that demand takes high or low values with high probabilities. To understand the intuition, consider the extreme case where $k = 1/2$. If the firm follows the OR strategy, then we expect the optimal order quantity to be $(Q_r, Q_o) =(\mu - \delta_r, \mu + \delta_o)$. On the other hand, if the firm follows the RO strategy, then the optimal order quantity would be $(Q_r, Q_o) = (\mu + \delta_r, \mu - \delta_o)$. In both strategies, the transshipment quantities are $\min(2\delta_r, 2\delta_o)$ with probability 0.25. However, the local sales are different under the two strategies. Specifically, the expected local sales is $2\mu - \delta_r$ for the OR strategy and the expected local sales is $2\mu - \delta_o$ for the RO strategy. Clearly, $2\mu - \delta_r > 2\mu - \delta_o$. So, the OR strategy dominates the RO strategy as predicted by Proposition 4.4.3.

Proposition 4.4.3 provides analytical insights in guiding a firm's channel integration strategy. In the next section, we test the robustness of these insights for normal and gamma demand distributions.

4.5 Computational Studies

In this section, we extend our analysis to normal and gamma demand distributions using numerical studies. Specifically, we try to answer the following questions. 1. Are the insights developed in the previous section robust to demand distribution assumptions? 2. When does channel integration strategy matters? 3. What is the impact of the number of retail stores on the firm's channel integration strategy?

4.5.1 Robustness

In this section, we first study the optimal strategy when the channel demands D_r and D_o are normally distributed, and then discuss the case in which the channel demands are with gamma distributions. Let the mean and standard deviation of D_r (resp., D_o) are μ_r and σ_r (resp., μ_o and σ_o), respectively. Note that for normally distributed demands, a three-point distribution approximation will require a small k value. For example, when we use three-point distributions to approximate the standard normal distribution, we should set $k = 0.3035$ (Pflug 2001). Consequently, we provide the following guideline for channel integration strategy for normal demands based on Proposition 4.4.3.

Guideline 1: Suppose $p \geq 2c$. Then $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$ if $\sigma_r \leq \sigma_o$, and $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$ if $\sigma_r \geq \sigma_o$. Suppose $p < 2c$. Then $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$ if $\sigma_r \leq \sigma_o$, and $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$ if $\sigma_r \geq \sigma_o$.

We use computational studies to test the above guideline which can lead to the optimal strategy. Our computational study starts with a base-case scenario with $c = 7$, $p = 14$, $\tau = 3$, $\mu_o = \mu_r = 20$, $\sigma_o = \sigma_r = 8$ and $\rho = 0$, where ρ is the correlation coefficient between the two channel demands. Subsequently, additional scenarios are generated by varying one of the problem parameters: the product price p, the standard deviation σ_r of the retail store demand, the transshipment cost τ, and the demand correlation coefficient ρ. Specifically, we vary σ_r from 4 to 16, the retail price p from 8 to 25, and the transshipment cost τ from 0 to 4, all in steps of 1. We also vary ρ from -0.5 to 0.5 in steps of 0.1. For each test instance generated, we first compute the optimal order quantities for both integration strategies by solving the first-order conditions using the built-in functions of Matlab. Then, the built-in functions of Matlab are used to compute the expected profits of the two integration strategies. Finally, we compare the resulting optimal profits of the two strategies to obtain the optimal strategy.

Among all 12870 cases we have considered, the optimal strategies are consistent with our guideline. Figures 4.2, 4.3, 4.4 summarize our computational results. The vertical axis in all these figures represents

$$Percentage\ of\ Profit\ Increment = \frac{\pi^{\mathrm{RO}} - \pi^{\mathrm{OR}}}{\pi^{\mathrm{OR}}} \times 100.$$

Consequently, *RO system is optimal* when the Percentage of Profit Increment is *positive*; otherwise, OR system is optimal.

Figure 4.2 illustrates how the percentage of profit increment changes with respect to the demand uncertainty ratio (σ_r/σ_o) and the retail price. We can see that, consistent with our

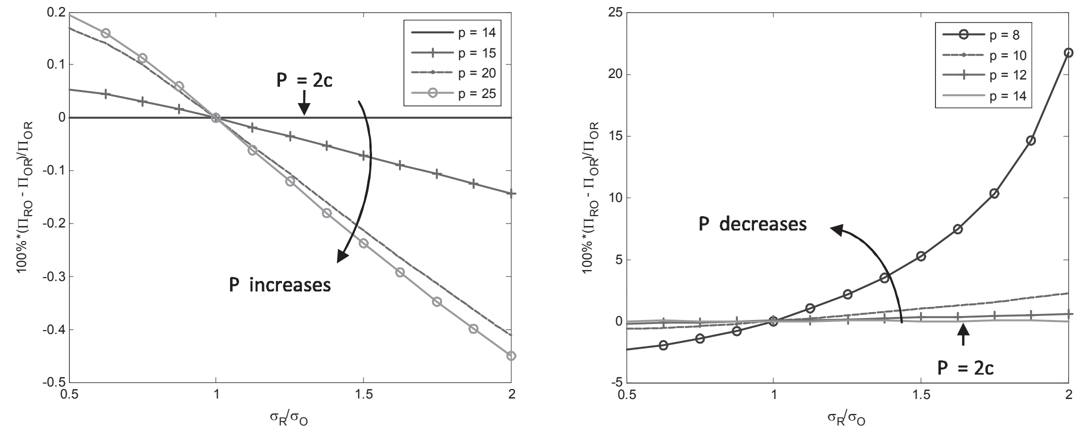


Figure 4.2. Percentage of Profit Increment $(100\times(\pi^{\mathrm{RO}}-\pi^{\mathrm{OR}})/\pi^{\mathrm{OR}})$ vs. Demand Uncertainty Ratio (σ_r/σ_o) for Different Retail Prices $(c=7, \tau=3, \mu=20, \sigma_o=8, \rho=0)$: (a) $p \geq 2c$; (b) $p \leq 2c$

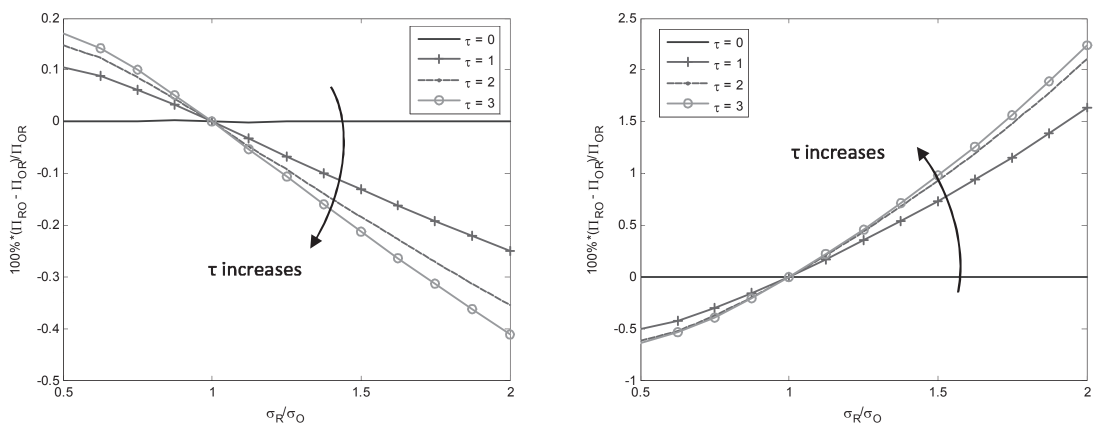


Figure 4.3. Percentage of Profit Increment $(100\times(\pi^{\mathrm{RO}}-\pi^{\mathrm{OR}})/\pi^{\mathrm{OR}})$ vs. Demand Uncertainty Ratio (σ_r/σ_o) for Different Transshipment Costs $(c=7, \mu=20, \sigma_o=8, \rho=0)$: (a) $p=20$; (b) $p=10$

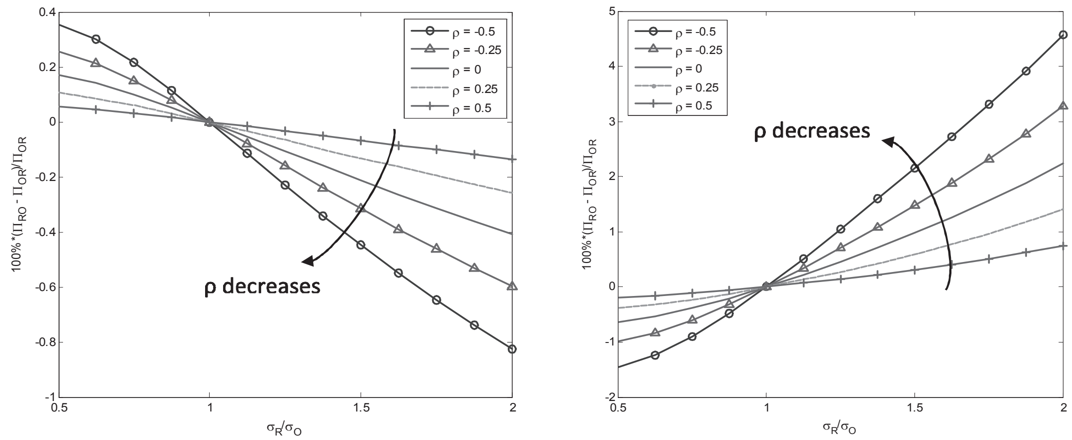


Figure 4.4. Percentage of Profit Increment $(100\times(\pi^{\mathrm{RO}}-\pi^{\mathrm{OR}})/\pi^{\mathrm{OR}})$ vs. Demand Uncertainty Ratio (σ_r/σ_o) for Different Demand Correlation Coefficients $(c = 7,\ \mu = 20, \sigma_o = 8, \tau = 3)$: (a) $p = 20$; (b) $p = 10$

guideline, for high-margin products (Figure 4.2a), RO system dominates OR system when $\sigma_r < \sigma_o$ while OR system is better when $\sigma_r > \sigma_o$. For low-margin product (Figure 4.2b), it is the other way around. Figure 4.3 (resp., Figure 4.4) illustrates how the percentage of profit increment changes with respect to the demand uncertainty ratio (σ_r/σ_o) and the transshipment cost τ (resp., the demand correlation coefficient ρ) for high-margin and low-margin products. Consistent with what we observe in Figure 4.2, regardless of the values of τ and ρ, for high-margin products, the firm should use the leftover inventory from the channel with low demand uncertainty to feed the other channel, while it is the reverse for low-margin products.

We repeat all the computational experiments above with gamma demand distributions. To find the optimal strategy with gamma demands, we first estimate the optimal expected profit for each strategy with Monte Carlo simulation, implemented in C++. To generate correlated gamma random variables, we use the approach by Damme (2010), which is based on Schmeiser and Lal (1982). Then, we obtain the optimal strategy by comparing the two resulting profits. From our computational studies, we see that the optimal strategy still depends on the product contribution margin, and that the transshipment direction when the margin is high is opposite to the transshipment direction when the margin is low. Owing to the fact that a gamma density is not symmetric, the critical value of the gross margin $(p - c)/p$ to differentiate the low-margin from high-margin products is not 50% as in the three-point or normal demand case. As a matter of fact, the critical value of the gross margin may be different for different problem instances. However, our insights regarding the optimal strategies remain valid: When the margin is high, the retailer should use the channel with low demand uncertainty to feed the channel with high demand uncertainty, and it is the other way around when the margin is low.

4.5.2 When Does Integration Strategy Matter?

Of course, channel integration comes with a cost. Therefore, besides the optimal integration itself, we would like to know when the choice of integration strategy makes a significant difference to the firm's profit. That is, when does integration strategy matter? From the definition of Percentage of Profit Increment, we know that when the absolute value of Percentage of Profit Increment increases, the firm can benefit more by adopting the appropriate strategy instead of the other.

We can see from Figure 4.2 that for high-margin products (Figure 4.2a), the difference between the two systems becomes larger when the price increases, while for low-margin products, the difference between the two systems becomes larger when the price decreases. As a matter of fact, when $c = 7$ and $p = 8$, the difference between the two systems can be as large as 20%. Thus, we conclude that the integration strategy is of consequence when the margin is very high or very low.

Figure 4.3 illustrates that, the integration strategy matters when the transshipment cost is high regardless of the margin. Obviously, with either integration strategy, the optimal profit decreases as the transshipment cost increases. However, from Figure 4.3 we can infer that with a high transshipment cost, a wrong integration strategy would deteriorate the firm's performance more than the right integration strategy would. In other words, when the two channels are integrated using a proper strategy, the optimal profit is less sensitive to the increase in the transshipment cost.

Figure 4.4 demonstrates how the coefficient of correlation between the two channel demands impacts the value of the optimal strategy for high-margin products (Figure 4.4a) and low-margin products (Figure 4.4b). As shown in these two figures, regardless of the margin, the optimal strategy yields higher benefit with strongly negatively correlated demands between the two channels. Intuitively, when demands are strongly positively correlated, the benefit from risk pooling is low. Therefore, the gain from the right integration strategy is also expected to be low. In practice, demands in the two channels tend to be independent. (For example, based on the weekly sales data for HP printer at 178 retail stores and the HP Shopping Village, Seifert et al. (2006) report that the correlation between retail store sales and the HP Shopping Village sales is 0.0494.) From Figure 4.4 we can see that, for products whose demands tend to be fairly independent, the improvement from the right integration strategy can be significant, especially for low-margin products. Among all cases illustrated in Figures 4.2 to 4.4, the integration strategy matters when the difference between the two channel demand uncertainties is large.

In summary, the channel integration strategy would make a significant impact on the firm's profit when the product contribution margin is very high or very low, when the transshipment cost is high, when demand correlation between the two channels is low, or when the difference between the two channel demand uncertainties is large. Our computational studies for gamma demand distributions also reveal similar insights.

4.5.3 Integration Strategy with Multiple Retail Stores

In practice, retailers usually operate multiple retail stores. In this section, we examine how the number n of retail stores affects the channel integration strategy. To facilitate the computational study, we assume that there is no transshipment between the retail stores. We discuss the case with transshipment between the retail stores in Section 4.7.

As n increases, the "total demand uncertainty" in the physical channel increases. Based on Proposition 4.4.3 and the insights developed in Section 4.5.1, we provide the following guideline for the optimal strategy regarding the impact of the retail store number.

Guideline 2: For high-margin products, the firm should adopt the site-to-store strategy when the number of retail stores is large. For low-margin products, the firm should use the store-to-site strategy when the number of retail stores is large.

To test our guideline, we first conduct a series of numerical experiments for normal demands. In our computational study, we assume that the demand D_{ri} $(i = 1, 2, ..., n)$ in different retail stores are identically distributed with mean μ_r and standard deviation σ_r. The coefficients of correlation between the online demand and each retail store's demand are identical as ρ. We fix $c = 6$, $\mu_o = 50$, $\sigma_o = 16$, $\mu_r = 12$, $\sigma_r = 4$, $\tau = 1$, $\rho = 0$, $p = 13$ or 8, and increase the number of retail stores from 2 to 20 in steps of one to generate 38 instances. Figure 4.5 summarizes the results and illustrates how the optimal strategies for high-margin products (Figure 4.5a) and low-margin products (Figure 4.5b) change as the number of retail stores increases. We can see from these figures that, these results are consistent with our guideline. As the number of retail stores increases (more retail stores are open), the total uncertainty in the retail channel becomes large. The company should switch from the store-to-site strategy to the site-to-store strategy for high-margin products. An example in practice is that of Adidas, which adopts the site-to-store strategy for selling high-margin products in numerous retail stores.

In addition, as the number of retail stores increases, the company can benefit more by choosing the correct integration strategy for both high-margin and low-margin products. This implies that the integration issue is more important for a nationwide retailer with more retail stores than a local retailer with fewer stores.

When there is only one retail store, we have a clear and simple rule to decide which integration strategy is optimal under all kinds of scenarios. However, in the case of multiple retail stores, the rule is not as clear. Hence, we develop a heuristic based on Proposition

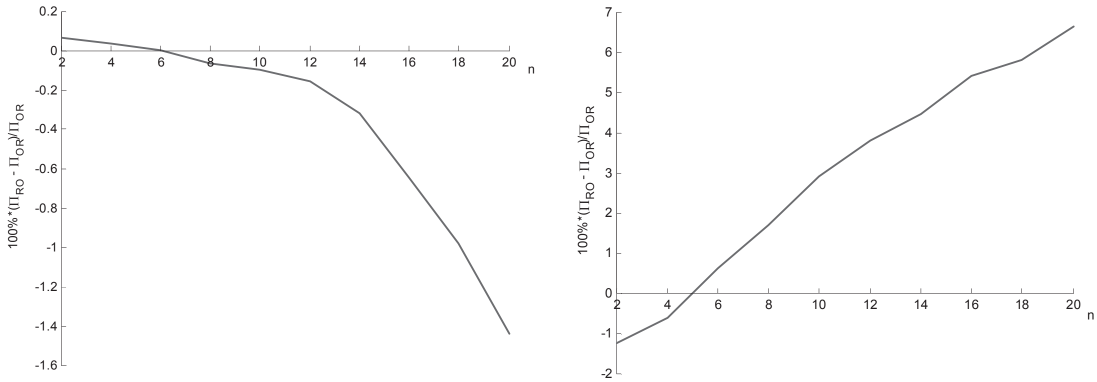


Figure 4.5. Percentage of Profit Increment $(100 \times (\pi^{\mathrm{RO}} - \pi^{\mathrm{OR}})/\pi^{\mathrm{OR}})$ vs. Number of Retail Stores (n) $(c = 6, \mu_o = 50, \sigma_o = 16, \mu_r = 12, \sigma_r = 4, \tau = 1, \rho = 0)$: (a) $p = 13$; (b) $p = 8$

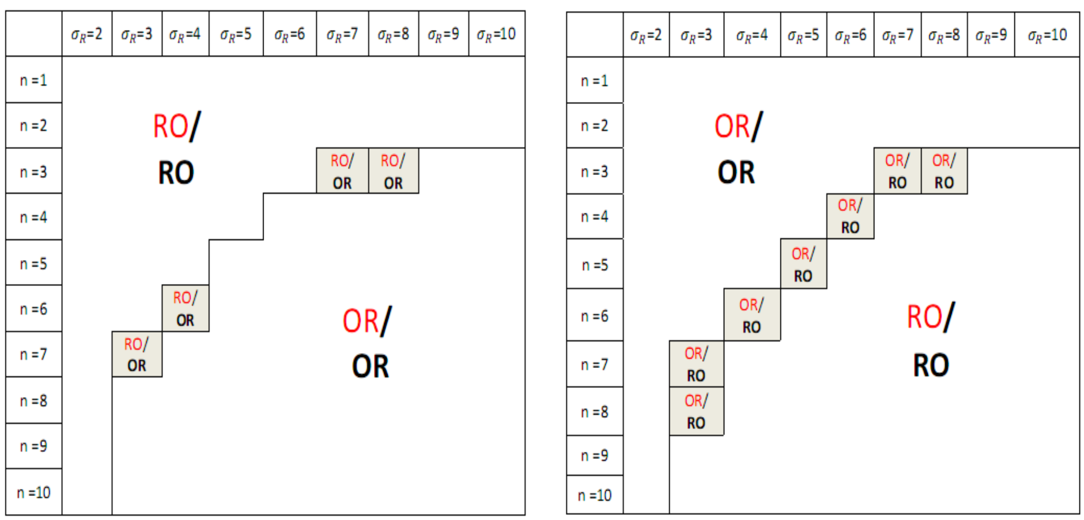
4.4.3. To this end, we first consider a retailing system consisting of n identical newsvendors with purchasing cost c and retail price p. It can be demonstrated that when there is no transshipment between the newsvendors, such a system of retail stores is equivalent to a newsvendor whose demand is normal with mean $n\mu_r$ and standard deviation $n\sigma_r$ (Zhang 2005). This implies that the integration strategy of the multiple-retail-store case can be approximated by the integration strategy of the one-retail-store case. Therefore, we prescribe the following guideline to identify an effective channel integration strategy for the multiple-retail-store case.

Guideline 3: Suppose $p > 2c$. Then the firm should use store-to-site if $n\sigma_r < \sigma_o$, and use site-to-store if $n\sigma_r > \sigma_o$. Suppose $p < 2c$. Then the firm should use site-to-store if $n\sigma_r < \sigma_o$, and use store-to-site if $n\sigma_r > \sigma_o$.

To test the effectiveness of the heuristic guideline, we have conducted an extensive computational study. In the computational study, we fix $c = 6$, $\tau = 1$, $\mu_o = 100$, $\sigma_o = 20$, $\mu_r = 30$ and $\rho = 0$. We vary the number n of retail stores from 1 to 10 and the standard deviation σ_r of the demand in each retail store from 2 to 10, both in steps of 1 to obtain 90 problem instances. We choose the prices to be 13 to examine the high-margin product case and 8 for the low-margin product case. Altogether, we identify the optimal and heuristic integration strategies for 180 problem instances. The optimal strategy is found by using Monte Carlo simulation, and the heuristic integration strategy is attained by applying the simple Guideline 3.

Table 4.1(a) for $p = 13$ ($p > 2c$), and Table 4.1(b) for $p = 8$ ($p < 2c$) summarize our computational results. Each table consists of 90 cells. The cells with gray background are the instances in which the optimal and heuristic integration strategies differ. Within these cells, the light face system denotes the optimal strategy while the bold face system denotes the heuristic strategy. For the problem instances corresponding to the other cells, the heuristic strategy is identical to the optimal strategy, as specified in the tables.

Table 4.1. Optimal Strategy (light face) vs. Heuristic Strategy (bold face): (a) $p > 2c$; (b) $p < 2c$



From both tables, we see that the heuristic yields the optimal strategy in most instances. Specifically, the heuristic strategy is optimal in 86 (resp., 83) out of 90 instances for high-margin (resp., low-margin) products. The instances in which the optimal and the heuristic strategies differ are close to the diagonal cells, where the demand uncertainties in the two channels are close. However, the optimal strategy does not affect the firm's profit significantly in these cases. As a matter of fact, the average loss incurred from choosing a wrong strategy in these cases is only 0.01% for high-margin products and 0.08% for low-margin products. We expect that the heuristic will perform even better as the number of retail stores increases. This is because as the number of retail stores increases, the difference in the demand uncertainties between the two channels becomes larger. Simulations with other parameter combinations (i.e., repeating the simulation by varying the value of p, c, τ, or σ_r, one at a time) as well as with correlated demands (we keep the correlation coefficients between any two stores to be $\pm 0.1/n$, $\pm 0.3/n$, or $\pm 0.5/n$) yield similar results. Therefore, the retailer can apply the guideline prescribed above to the multiple-retail-store case without incurring a significant loss.

Besides the heuristic guideline proposed above, another heuristic is to compare the expected demands of the two channels. However, from our earlier analysis, we know that the demand averages do not play a role in determining the integration strategy. This means that it is possible to have a totally different (wrong) result by simply comparing the expected store demands.

4.6 Application: A Spatial Model of Dual-Channel Retailing Systems

In this section, we first propose a modified circular spatial model to specialize the demands of the online store and each retail store in a dual-channel retailing system. The dual-channel retailing system is modeled with a circle of unit circumference, n retail stores and one online store. The n retail stores are evenly distributed on the circumference of the circle and the online store is located at the center of the circle; see Figure 4.6.

The total number of customers is a fixed number m; later in this section, we extend our model to allow the total number of customers to be a random variable. A customer can purchase at most one unit of the product. So, each customer has three options: purchasing one unit from one of the retail stores, purchasing one unit from the online store, or purchasing nothing; she will choose the option that provides the highest utility. If a customer buys the product from a retail store, she has to travel to the physical store. As in standard circular models, we assume that a customer incurs a cost t for one unit of travelling distance. If

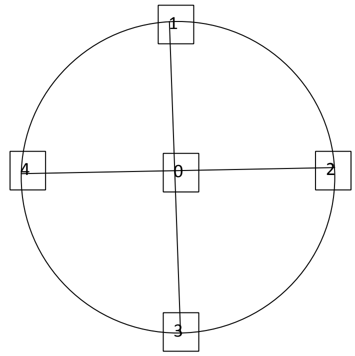


Figure 4.6. An Illustration of the Circular Model with Four Retail Stores

a customer buys the product from the online store, an inconvenience cost is incurred by the customer. This inconvenience cost is due to the lack of immediate gratification, lack of physical inspection, or difficulty in product return, when compared with an in-store purchase. We denote the inconvenience factor as k. Let v be the utility from consuming the product and, as before, let p be the unit price.

We assume that, when purchasing in a retail store, the customer buys the product from the closest physical store. Therefore, for a customer located at a distance d from the closest physical store ($0 \leq d \leq \frac{1}{2n}$), her utility surpluses of buying in-store and online can be expressed, respectively, as $U_r(d) = u_r(d) + \epsilon_r$ and $U_o = u_o + \epsilon_o$, where $u_r(d) = v - p - d \times t$ and $u_o = v - p - k$ are her respective nominal (expected) utility surpluses of buying in-store and online, and ϵ_i $(i = r, o)$ is a Gumbel random variable. ϵ_i's distribution is $\Pr(\epsilon_i \leq x) = \exp(-e^{-((x/\mu)+\gamma)})$ with mean zero, where γ is Euler's constant ($\gamma \approx 0.5772$) and μ is a positive constant. A higher μ implies a higher degree of heterogeneity among the customers. Without loss of generality, we assume that the expected utility of no purchase is zero. Assume that each customer has an equal probability located at any place on the circle and each customer's purchasing decision is independent of the others. Let b_{ri}, b_o, and b_{null} denote the probabilities of purchase from the ith retail store ($1 \leq i \leq n$), from the online store, and no purchase, respectively. Following the standard analysis of the multinomial logit (MNL) model (see, for example, Anderson et al. 1992, p.39-40) and circular model (see, for example, Tirole 1992, p.282-285), we have

$$
\begin{aligned}
b_{ri} &= \frac{2\mu}{t} \ln \left(\frac{\exp((v-p)/\mu) + \exp((v-p-k)/\mu) + 1}{\exp((v-p-\frac{t}{2n})/\mu) + \exp((v-p-k)/\mu) + 1} \right), \\
b_o &= \frac{\exp((v-p-k)/\mu)}{\exp((v-p-k)/\mu) + 1}(1 - nb_{ri}),\ b_{null} = 1 - nb_{ri} - b_o.
\end{aligned}
$$

Since $b_{r1} = b_{r2} = ... = b_{rn}$, we simply term them as b_r.

Aggregating over all customers, the demand for each individual retail store or the online store follows a binomial distribution with parameters m and either b_r or b_o. When the market size is large enough, each of these binomial distributions can be approximated by a corresponding normal distribution. Therefore, the demand for the ith retail store D_{ri} and the demand for the online store D_o are approximately normal random variables with

$$
\begin{aligned}
D_{ri} &\sim N(mb_r, \sqrt{mb_r(1-b_r)}),\ D_o \sim N(mb_o, \sqrt{mb_o(1-b_o)}), \\
\rho_{D_{ri}D_o} &= -\sqrt{\frac{b_r b_o}{(1-b_r)(1-b_o)}}, \rho_{D_{ri}D_{rj}} = -\frac{b_r}{1-b_r}(i \neq j).
\end{aligned}
$$

The circular model has been widely used in the literature for the market of differentiated products since the seminal work of Salop (1979). Balasubramanian (1998) was the first to model a dual-channel retailing system with the circular model by incorporating a direct channel at the center of the circle. Our model follows Balasubramanian (1998) and captures the following important features of dual-channel retailing systems with a physical channel and an online channel. First, the physical channel consists of multiple retail stores and the online channel consists of only one online store. Second, the travelling distance to a retail store is a dominant factor that influences the patronage of a customer. Third, online shopping incurs an inconvenience cost for a customer due to, for example, the difficulty of physical inspection and slow shipping; see Forman et al. (2009) for empirical studies on how the trade-off between the offline transportation cost and the online disutility cost determines customer choice of channels. Lastly, all customers have equal access to the online store. Our model is different from Balasubramanian (1998) in several ways. The most important one is that to model customer heterogeneity, we use the MNL random utility model; as a result, demands are uncertain in our model. To our knowledge, Chen et al. (2008) is the only paper that studies a dual-channel system with demand uncertainty that is built on a customer choice model. In their model, demands in the two channels are perfectly positively correlated, whereas our model allows for more flexibility in modeling the demand correlation.

Channel Integration Strategy. According to the result in Section 4.5.3, one should compare p and $2c$, as well as $n\sigma_r = n\sqrt{mb_r(1-b_r)}$ and $\sigma_o = \sqrt{mb_o(1-b_o)}$, to decide the preferred integration strategy. Specifically,

Guideline 4: For $p > 2c$, the preferred strategy is store-to-site if $n\sqrt{b_r(1-b_r)} < \sqrt{b_o(1-b_o)}$ and site-to-store, otherwise. For $p < 2c$, the preferred strategy is site-to-store if $n\sqrt{b_r(1-b_r)} < \sqrt{b_o(1-b_o)}$ and store-to-site, otherwise.

In the following analysis, we use high-margin products (with $p > 2c$) as an example, and study the effects of the travelling cost t, the online inconvenience factor k, and the number of retail stores n on the channel integration strategies. Similar analysis can be made for low-margin products.

The case with $n = 1$ is quite straightforward. In the following analysis, we assume $n \geq 2$ unless otherwise specified. Because $nb_r + b_o \leq 1$, we have $b_r \leq 1/2$ if $n \geq 2$. We can also safely assume $b_o \leq 1/2$, considering the relatively small portion of online sales. Notice that the function $x(1-x)$ increases in x when $x \leq 1/2$. Therefore, $n\sigma_r$ (resp., σ_o) increases in b_r (resp., b_o).

Impact of online-shopping inconvenience factor. It is easy to see that b_o increases and b_r decreases as the online-shopping inconvenience factor k decreases. Therefore, as the retailer improves customers' online shopping experience by, for example, redesigning the web site, offering more return options, or offering free shipping, $\sqrt{b_o(1-b_o)}$ will increase and $n\sqrt{b_r(1-b_r)}$ will decrease. Based on our channel integration guideline, we have the following result:

> A retailer following the store-to-site strategy should continue following the same strategy as the inconvenience factor decreases. On the contrary, a site-to-store strategy follower should watch the relationship between $\sqrt{b_o(1-b_o)}$ and $n\sqrt{b_r(1-b_r)}$ closely as the inconvenience factor is improved.

Impact of unit travelling cost. Similarly, as the unit travelling cost t decreases, customers are more willing to visit and then buy from physical stores. This implies a higher b_r and a lower b_o, and therefore a higher $n\sqrt{b_r(1-b_r)}$ and a lower $\sqrt{b_o(1-b_o)}$. Consequently, we have the following result:

> As the travelling cost decreases, a site-to-store strategy follower can stay with his channel integration strategy while a store-to-site strategy follower should be ready to switch to the alternative strategy.

Impact of number of retail stores. As more retail stores are opened, customers on average have a shorter travelling distance, and then can save on travelling cost. Hence, the expected demand of the online channel decreases. The expected demand for individual physical stores also decreases, but the expected total demand for the whole physical channel increases. That is, b_o (and hence $\sqrt{b_o(1-b_o)}$) and b_r decrease in n, while nb_r increases in n. Because $n\sqrt{b_r(1-b_r)} = \sqrt{n \times nb_r \times (1-b_r)}$, we know that $n\sqrt{b_r(1-b_r)}$ increases in n. Therefore, we have the following result:

> The site-to-store strategy is attractive for a nationwide retailer with many retail stores.

Extension to random market size. We have specialized our main results to a model of dual-channel retailing systems that captures important differences between shopping online and offline. In our analysis above, we assume that the potential market size is fixed.

Our analysis can be extended to the case in which the market size is a random variable M with $E[M] = \mu_M$ and $Stdev[M] = \sigma_M$. With the earlier results for a given market size m and standard formulas for conditional probability, we can for the random market size M, obtain $E[D_{ri}] = \mu_M b_r$, $Stdev[D_{ri}] = \sqrt{\mu_M b_r - b_r^2(\mu_M - \sigma_M^2)}$, $E[D_o] = \mu_M b_o$, $Stdev[D_o] = \sqrt{\mu_M b_o - b_o^2(\mu_M - \sigma_M^2)}$, $Cov(D_{ri}, D_o) = -b_r b_o(\mu_M - \sigma_M^2)$, $Cov(D_{ri}, D_{rj}) = -b_r^2(\mu_M - \sigma_M^2)$, $\rho_{D_{ri}D_o} = -\frac{b_r b_o(\mu_M - \sigma_M^2)}{\sqrt{\mu_M b_r - b_r^2(\mu_M - \sigma_M^2)}\sqrt{\mu_M b_o - b_o^2(\mu_M - \sigma_M^2)}}$ and $\rho_{D_{ri}D_{rj}} = -\frac{b_r^2(\mu_M - \sigma_M^2)}{\mu_M b_r - b_r^2(\mu_M - \sigma_M^2)}$. Here, we can see that the demand correlation between the physical and online channels can be positive, zero, or negative, depending on the relation between μ_M and σ_M^2. Therefore, our circular model is very flexible in modeling the demands of dual-channel retailing systems. From our results in §4.5, the demand correlation will not change the basic integration guideline. Therefore, retailers should again choose the integration strategy according to the product contribution margin and standard deviations of both channel demands.

4.7 Conclusion

Although it is well known that a retailer can benefit from dual-channel integration, less is known about how to integrate a physical channel and an online channel effectively. In this chapter, we present an analytical model to study channel integration decisions in a dual-channel retailing system by focusing on two standard integration strategies: store-to-site and site-to-store, and compare their performance under different scenarios. Our model belongs to the broad class of newsvendor networks but has a distinct focus on the network design in the presence of interchannel one-way transshipment. We contribute to the newsvendor network literature by studying the optimal network design problem under three-point demand distributions. Via our numerical studies, we show that the insights developed under three-point demand distributions continue to hold for normal and gamma demand distributions.

We provide a simple heuristic to identify an effective integration strategy for the multiple-retail-store case. The heuristic only requires evaluating the standard deviation of the demand in the online channel and the total of the standard deviations of all the retail stores' demands in the physical channel. Note that our heuristic is based on the result that an n-retail-store system (with no transshipment between the stores) is equivalent to a one-retail-store system. When transshipment is allowed between the retail stores, such an equivalence result can also be established by adjusting the demand of the equivalent retail store. Consequently, our heuristic can be extended to such cases in a straightforward way.

Finally, we propose a circular spatial model for dual-channel retailing systems. The model captures main drivers for customer purchasing behavior, such as the travelling distance to a retail store and the inconvenience of shopping online. We demonstrate how our results can be applied to these retailing systems and develop insights on how customer purchasing behavior drivers impact a retailer's channel integration strategy.

In our model, the optimal strategy is developed by examining the demand characteristics and contribution margin of one product. In practice, a retailer sells thousands of products rather than only one product. However, our analysis should help managers to understand how the two key factors (product contribution margin and channel demand distribution shape) affect the right channel integration strategy. In addition, we expect that our results will continue to hold under the multiple-product case, when we consider the average margin and the distribution shape of the aggregated demand among all the products.

CHAPTER 5

CONCLUSION AND DIRECTION FOR FUTURE RESEARCH

This chapter concludes the findings of this research, and suggests directions for future research.

In Chapter 2, we study how a monopolistic firm chooses between two product rollover strategies, single rollover and dual rollover, when selling to a market composed of both strategic and myopic customers. The important managerial insights from our analysis are:

- The lower the innovation is, the more valuable single rollover becomes. As the number of strategic customers increases, single rollover becomes more attractive.

- With low and medium innovation, the firm can eliminate strategic waiting behavior completely by committing to single rollover, while with high innovation, waiting incentive may be lessened but not completely eliminated.

- With low or medium innovation, a firm following dual rollover should be ready to switch to single rollover, especially when the disposal value of the leftover old version under single rollover and the proportion of strategic customers are not very low. Furthermore, such a firm should search for a high value disposal option.

- With high innovation, a firm can still increase its profit by adopting single rollover when the proportion of strategic customers is high and the disposal value of the leftover old version under single rollover is *low*. Therefore, such a firm does not necessarily benefit from a high value disposal option. A potentially lowest value disposal option is donating the leftovers, and it can give the highest profit. Thus, profit-making objective can lead to socially-responsible outcomes.

- With high innovation, the firm can introduce both versions hoping that customers will purchase both of them. With low and medium innovations, however, if the innovation can compensate the firm's profit discounting and customers' value depreciation over time, the firm should skip the old version in order to eliminate the cannibalization for the new version; otherwise, as long as customers value the current (old) version, the firm should introduce it as soon as possible to avoid the loss from time depreciation.

According to Saunders and Jobber (1994), slightly more than half of the companies use some sort of dual rollover strategy. Erhun et al. (2007) point out that dual rollover is widely used in the high-tech industry. The managerial insights above can help companies to decide whether they should switch to single rollover. Our numerical study shows a broad application potential and significant profit increase with single rollover compared with dual rollover. These application areas broaden further as innovation rate drops with frequent product introductions and/or as more customers are better informed and become strategic.

In Chapter 3, we investigate the impact of rollover strategies (single and dual rollovers) and customer behavior (strategic and myopic behaviors) on a firm's innovation level and profit. The most interesting finding is that strategic behavior speeds up a firm's innovation process, which is different from the common wisdom and the extant marketing literature. Table 5.1 below summarizes our main results:

Table 5.1. Comparison of Profit and Innovation in Four Settings

$\Pi^{\text{M-DR}}$	$\geq$	$\Pi^{\text{M-SR}}$	$\theta^{\text{M-DR}}$	$\leq$	$\theta^{\text{M-SR}}$
$\geqslant$		$=$	$\leq$		$=$
$\Pi^{\text{S-DR}}$	$\geq$ or $\leq$	$\Pi^{\text{S-SR}}$	$\theta^{\text{S-DR}}$	$\leq$ or $\geq$	$\theta^{\text{S-SR}}$

In this chapter, we study the impact of strategic waiting behavior on innovation level. Besides the strategic waiting, customers may have other features/behaviors, such as the exclusivity-seeking behavior as discussed in Toktay et al. (2011). It will be interesting to study how the other customer behaviors impact a firm's innovation path and how the different rollover strategies influence the other customer behaviors.

In Chapter 4, we compare two typical dual-channel integration strategies: site-to-store and store-to-site. Table 5.2 summarizes our prescription for choosing an appropriate channel integration strategy when k is small. In the table, Channel A –> Channel B means that the leftover inventory in Channel A is used to fulfill the unmet demand in Channel B. With a large value of k, the direction of transshipment stays the same when inventory is stored in only one channel, while the direction of transshipment reverses when inventory is kept in both channels.

Table 5.2. Summary of Channel Integration Guidelines When k is Small

	Direction of Transshipment	
	Inventory in one channel	Inventory in both channels
High-margin products	Channel with stochastically larger or less uncertain demand –>	Channel with lower demand uncertainty –> Channel with higher demand uncertainty
Low-margin products	Channel with stochastically smaller or more uncertain demand	Channel with higher demand uncertainty –> Channel with lower demand uncertainty

Additionally, our results provide the following important managerial insights: (1) As the number of retail stores increases, the site-to-store (resp., store-to-site) strategy becomes more attractive for high-margin (resp., low-margin) products with normal channel demands. (2) Managers should pay particular attention to integration strategies when facing one or more of the following situations under which the optimal strategy makes a significant difference: (i) the product contribution margin is either very high or very low, (ii) the transshipment cost is high, (iii) the demand correlation between the two channels is not strongly positive, and (iv) the demand uncertainty difference between the two channels is large.

We assume that product demand is not affected by the channel integration strategy and store demand is not a function of the product inventory level. This assumption holds when customers are not strategic. It also holds when the products are infrequently purchased or less promoted. For other products, the channel integration strategy or the inventory levels may have an impact on channel demands by affecting customers purchasing pattern (see, for example, Smith and Achabal 1998, and Chen et al. 2008). For instance, the two channels when integrated may cannibalize each other or expand the whole market size. It would be interesting to see how the optimal strategy would be different when such an impact is considered.

CHAPTER 6

PROOFS

6.1 Proofs related to Chapter 2

Proof of Proposition 2.3.1

With more than one version and customer segment in period 2, different versions can be targeted to different segments. We first list all possible targeting strategies, and then list the highest prices that can be used to execute each targeting strategy and consequential stocking levels. After that, we investigate the conditions under which a certain strategy (involving both prices and stocking levels) in period 2 is most profitable in two cases $[p_1 = R_f]$ and $[p_1 = v]$. Finally, we compute the expected waiting surpluses in each of the two cases.

We use B to denote bargain hunters and use a vector notation to denote a targeting strategy; the first (second) entry in the vector denotes the customer segments targeted by version V1 (V2) in period 2. For example, $[\{\text{B}\}, \{\text{P1B,P1NB}\}]$ means targeting B with V1 and both P1B and P1NB with V2. Under each targeting strategy, we also discuss P1NB's preference between V1 and V2. With low innovation, the firm's all possible targeting strategies are: $[\{\text{P1NB}\}, \{\text{P1NB}\}]$, $[\{\text{B}\}, \{\text{P1NB}\}]$, $[\{\text{B,P1NB}\}, \{\text{P1NB}\}]$.

The highest prices to execute $[\{\text{P1NB}\}, \{\text{P1NB}\}]$ **are high-high prices:** $p_1' = \beta v$**,** $p_2 = \beta v(1+\theta)$, which are the P1NB's respective reservation prices for V1 and V2. With $\beta v(1+\theta) - c < \beta v$, the firm induces P1NB to buy V1 rather than V2. Because $\beta v(1+\theta) - c \geq 0$ and there is no uncertainty in period 2, the firm produces exactly $[(n-s)-(q_1-s)]^+ = (n-q_1)^+$ of V2 to meet all unsatisfied P1NB demand overflowed from V1. So, the firm's profit in period 2 is

$$\Pi_2^{D(H-H)} = [\beta v(1+\theta) - c](n-q_1)^+ + \beta v \min\{n-s, q_1-s\}. \tag{6.1}$$

$\Pi_2^{D(H-H)}$ denotes the firm's profit with H-H strategy in period 2. Similar naming conventions are used in the appendix.

The highest prices to execute $[\{B, P1NB\}, \{P1NB\}]$ **are low-high prices:** $p_1' = \delta$, $p_2 = \beta v(1+\theta)$, which are the B's reservation price for V1 and the P1NB's reservation price for V2, respectively. With the positive surplus $(\beta v - \delta)$ from V1 and zero surplus from V2, P1NB prefer V1 and switch to V2 only when V1 is out of stock. The remaining V1s (if any) after P1NB's purchase are sold to B. The profit is

$$\Pi_2^{D(L-H)} = [\beta v(1+\theta) - c](n - q_1)^+ + \delta(q_1 - s). \tag{6.2}$$

In order to execute $[\{B\}, \{P1NB\}]$, the firm needs to make V2 the P1NB's first option, which requires $\beta v(1+\theta) - p_2 \geq \beta v - \delta$, i.e., $p_2 \leq \beta v\theta + \delta$. However, $p_2 \leq \beta v\theta + \delta$ implies a negative margin from V2 with low innovation $\theta < (c-\delta)/(\beta v)$. So, $[\{B\}, \{P1NB\}]$ cannot be successfully executed.

$[p_1 = R_f]$: $s = \min\{q_1, n\}$. If $n \leq q_1$, then $s = n$. L-H strategy is better in view of (6.1)-(6.2) as $\Pi_2^{D(H-H)} = 0 < \delta(q_1 - n) = \Pi_2^{D(L-H)}$. If $n > q_1$, then $s = q_1$. From (6.1) and (6.2), we have $\Pi_2^{D(H-H)} = [\beta v(1+\theta) - c](n - q_1) = \Pi_2^{D(L-H)}$. Both strategies yield the same profit, because there is no V1 left from period 1 and thus the price of V1 is of no consequence. Without loss of generality, we take H-H as optimal. Finally, we compute the expected waiting surplus $w(q_1, R_f, 1)$. When $n > q_1$, the firm uses H-H strategy and leaves zero surplus to waiting customers. When $n \leq q_1$, with $p_2 = \beta v(1+\theta)$, the surplus from V2 is also zero. Because P1NB prefer V1 to V2 when $n \leq q_1$, according to (2.12) and (2.13), we have $w(q_1, R_f, 1) = \int_0^{q_1} \{\min\{\frac{q_1 - x}{x - x}, 1\}(\beta v - \delta) + 0\} f(x)dx + 0 = \int_0^{q_1} (\beta v - \delta) f(x)dx = (\beta v - \delta)F(q_1)$.

$[p_1 = v]$: $s = \min\{q_1, (1-\phi)n\}$. If $n \geq \frac{q_1}{1-\phi}$, then $s = q_1$. From (6.1) and (6.2), $\Pi_2^{D(H-H)} = \Pi_2^{D(L-H)}$ as in the case $[p_1 = R_f]$. If $q_1 < n < q_1/(1-\phi)$, then $s = (1-\phi)n$, $\Pi_2^{D(H-H)} = [\beta v(1+\theta) - c](n - q_1) + \beta v[q_1 - (1-\phi)n]$ and $\Pi_2^{D(L-H)} = [\beta v(1+\theta) - c](n - q_1) + \delta[q_1 - (1-\phi)n]$. Because $\Pi_2^{D(H-H)} > \Pi_2^{D(L-H)}$, H-H strategy is optimal. If $n \leq q_1$, then $s = (1-\phi)n$, $\Pi_2^{D(H-H)} = \beta v[1 - (1-\phi)]n$ and $\Pi_2^{D(L-H)} = \delta[q_1 - (1-\phi)n]$. We then have $\Pi_2^{D(H-H)} \leq \Pi_2^{D(L-H)} \iff \beta v[1-(1-\phi)]n \leq \delta[q_1-(1-\phi)n] \iff n \leq \frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1$. Since $\frac{\delta}{\beta v\phi+\delta(1-\phi)} < \frac{\delta}{\delta\phi+\delta(1-\phi)} = 1$, we have $\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1 < q_1$. Then, L-H strategy is optimal only for $n \leq \frac{\delta q_1}{\beta v\phi+\delta(1-\phi)}$. We can compute the expected waiting surplus as

$$\begin{aligned} w(q_1,v,0) &= \int_0^{\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1} \{\min\{\frac{q_1-(1-\phi)x}{x-(1-\phi)x},1\}(\beta v-\delta)+0\}f(x)dx+0 \\ &= \int_0^{\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1} (\beta v-\delta)f(x)dx = (\beta v-\delta)F(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1). \end{aligned}$$

In the first equality, when $x > \frac{\delta q_1}{\beta v\phi+\delta(1-\phi)}$, the firm uses H-H strategy, which leads to zero surplus to waiting customers. The second equality follows from $\min\{\frac{q_1-(1-\phi)x}{x-(1-\phi)x},1\} = 1$ when $x < \frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1 < q_1$. ■

Before we prove Proposition 2.3.2, we first need to prove Lemmas 6.1.1 and 6.1.2.

Lemma 6.1.1 (Low innovation) *With $p_1 = v$, the unique solution to REE conditions except for the firm's pricing optimality (2.11) is $\chi = 0$, $W_c = (\beta v-\delta)F(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1)$, $R_f = v-(\beta v-\delta)F(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1)$, and q_1 as defined in i) and ii) below:*
i) If $c+\alpha[\beta v(1+\theta)-c] < v$, then $q_1 > 0$ and it is the unique solution of $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1} = 0$.
ii) If $c+\alpha[\beta v(1+\theta)-c] \geq v$, then $q_1 = 0$.

Proof. From (2.6), (2.7), (2.8) and Proposition 2.3.1, with $p_1 = v$ and q_1, we have the values for χ, W_c, R_f. So we focus on characterizing q_1. Note that

$$\begin{aligned} \mathbf{E}[\Pi^{D,h}(q_1,v)] = \mathbf{E}[v\min\{(1-\phi)N,q_1\}-cq_1] \\ &+\alpha\int_0^{\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1}\{[\beta v(1+\theta)-c](x-q_1)^+ + \delta[q_1-\min\{(1-\phi)x,q_1\}]\}f(x)dx \\ &+\alpha\int_{\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1}^{\infty}\{[\beta v(1+\theta)-c](x-q_1)^+ \\ &+\beta v[\min\{x,q_1\}-\min\{(1-\phi)x,q_1\}]\}f(x)dx, \end{aligned} \tag{6.3}$$

where the first term $\mathbf{E}[v\min\{(1-\phi)N,q_1\}-cq_1]$ is the expected profit in period 1 and the last two terms together are the expected profit in period 2 by using the pricing and stocking level values according to Proposition 2.3.1. We have $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1} = (v-\alpha[\beta v(1+\theta)-c])\overline{F}\left(\frac{q_1}{1-\phi}\right) - c + \alpha\delta F\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right) + \alpha(c-\beta v\theta)\left[F\left(\frac{q_1}{1-\phi}\right) - F(q_1)\right]$, $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1}\Big|_{q_1=0} = v - c - \alpha[\beta v(1+\theta) - c]$, $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1}\Big|_{q_1=\infty} = \alpha\delta - c < 0$ and $\frac{\partial^2 \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1^2} = f(q_1)\ [\frac{\alpha\delta^2}{\beta v\phi+\delta(1-\phi)}\frac{f\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right)}{f(q_1)} - \frac{v(1-\alpha\beta)}{1-\phi}\frac{1}{\frac{f(q_1)}{f(q_1/(1-\phi))}} - \alpha(c-\beta v\theta)]$. From the MSLR property, $f(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1)/f(q_1)$ as well as $f(q_1)/f(q_1/(1-\phi))$ are monotone in q_1 in the same direction. So

$$\text{if } \frac{\partial^2 \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1^2} \text{ crosses 0, it crosses at most once.} \tag{6.4}$$

By combining (6.4) and $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1}\Big|_{q_1=\infty} < 0$, we know that if $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1}\Big|_{q_1=0} = v - c - \alpha[\beta v(1+\theta) - c] > 0$, then $\mathbf{E}[\Pi^{D,h}(q_1,v)]$ must be unimodal. Moreover, there exists a unique positive root achieving $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1} = 0$, and this unique positive root q_1 maximizes $\mathbf{E}[\Pi^{D,h}(q_1,v)]$. This proves claim i).

Next, we prove claim ii). Rewrite the derivative as $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1} = v-c-\alpha[\beta v(1+\theta)-c] - v(1-\alpha\beta)F\left(\frac{q_1}{1-\phi}\right) - \alpha(c-\beta v\theta)F(q_1) + \alpha\delta F\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right)$. Because $\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1 < q_1 < \frac{q_1}{1-\phi}$, $\beta v\theta < c$, $\alpha < 1$ and $\beta < 1$, we have $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1,v)]}{\partial q_1} < v - c - \alpha[\beta v(1+\theta) - c] - v(1 - \alpha\beta)F\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right) - \alpha(c-\beta v\theta)F\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right) + \alpha\delta F\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right) = v - c - \alpha[\beta v(1+\theta) - c] - \{v - \alpha[\beta v(1+\theta) - c + \delta]\}F\left(\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1\right) < 0$. The last inequality follows from $v - c - \alpha[\beta v(1+\theta) - c] \leq 0$ and $v - \alpha[\beta v(1+\theta) - c + \delta] > v - c - \alpha[\beta v(1+\theta) - c]$. So, $q_1 = 0$. ■

Lemma 6.1.2 (Low innovation) *With $p_1 = R_f$, the unique solution to REE conditions except (2.11) is $\chi = 1$, $W_c = (\beta v - \delta)F(q_1)$, and p_1, q_1 as defined in i) and ii) below:*
i) If $c + \alpha[\beta v(1+\theta) - c] < v$, then $p_1 > c + \alpha[\beta v(1+\theta) - c]$ and $q_1 > 0$, and they can be uniquely determined from the two equations: $p_1 = v - (\beta v - \delta)F(q_1)$ and $F(q_1) = \frac{p_1 - c - \alpha[\beta v(1+\theta) - c]}{p_1 - \alpha[\beta v(1+\theta) - c + \delta]}$.
ii) If $c + \alpha[\beta v(1+\theta) - c] \geq v$, then $q_1 = 0$.

Proof. From (2.6), (2.7), (2.8) and Proposition 2.3.1, with $p_1 = R_f$ and q_1, we have the values for χ, W_c. From Proposition 2.3.1, we have $w(q_1, R_f, 1) = (\beta v - \delta)F(q_1)$, which together with (2.7) and (2.8) yields $R_f = v - (\beta v - \delta)F(q_1)$. So, according to (2.6), (2.10), and $R_f = v - (\beta v - \delta)F(q_1)$, we can obtain q_1 and p_1 by solving the simultaneous equations

$$\begin{cases} q_1 &= \arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1, R_f)], \\ p_1 &= R_f = v - (\beta v - \delta)F(q_1), \end{cases} \tag{6.5}$$

where $E[\Pi^{D,l}(q_1, R_f)]$ is the firm's expected total profit and

$$\begin{aligned} \mathbf{E}[\Pi^{D,l}(q_1, R_f)] &= \mathbf{E}[R_f \min\{N, q_1\} - cq_1] + \alpha\delta \int_0^{q_1} (q_1 - x)f(x)dx \\ &\quad + \alpha[\beta v(1+\theta) - c] \int_{q_1}^{\infty} (x - q_1)f(x)dx. \end{aligned} \tag{6.6}$$

By treating the right-hand sides in (6.5) as functions of p_1 and q_1, we have

$$q_1(p_1) = \arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1, p_1)], \tag{6.7}$$

$$p_1(q_1) = v - (\beta v - \delta)F(q_1). \tag{6.8}$$

We prove claims i) and ii) in three steps. *Step 1:* Optimal q_1 in (6.7) is continuous and non-decreasing in p_1 as in Lemma 6.1.3, which is stated and proved below. *Step 2:* From (6.8), we see that p_1 decreases in q_1. Also, $p_1 = v$ when $q_1 = 0$; $p_1 = v(1-\beta) + \delta$ when $q_1 = \infty$; and $v(1-\beta) + \delta \le p_1 \le v$. *Step 3:* From *Steps 1-2*, there must be a crossing point of (6.7) and (6.8). In addition, this crossing point is unique due to the non-decreasing property of $q_1(p_1)$ in (6.7) and the decreasing property of $p_1(q_1)$ in (6.8). ■

Lemma 6.1.3 *For a given p_1, there exists a maximizer $q_1^o(p_1) \ge 0$ of $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$. $q_1^o(p_1)$ satisfies:*

i)

$$q_1^o(p_1) = \begin{cases} 0 & \text{if } 0 < p_1 \le c + \alpha[\beta v(1+\theta) - c], \\ F^{-1}\left(\frac{p_1 - c - \alpha(\beta v(1+\theta) - c)}{p_1 - \alpha[\beta v(1+\theta) - c + \delta]}\right) & \text{if } p_1 > c + \alpha[\beta v(1+\theta) - c]. \end{cases}$$

ii) $q_1^o(p_1)$ *is continuous and non-decreasing in* p_1 *for all* $p_1 \geq 0$.

Proof. Similar to the case $p_1 = v$, we have $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = p_1\overline{F}(q_1) - c - \alpha[\beta v(1+\theta) - c]\overline{F}(q_1) + \alpha\delta F(q_1)$, $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2} = f(q_1)\{\alpha[\beta v(1+\theta)+\delta-c] - p_1\}$, $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=0} = p_1 - c - \alpha[\beta v(1+\theta) - c]$ and $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=\infty} = \alpha\delta - c < 0$. i) We investigate the maximizer $q_1^o(p_1)$ of $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ in three cases described by increasing p_1:

- $p_1 < \alpha[\beta v(1+\theta)+\delta-c]$: Since $c + \alpha[\beta v(1+\theta) - c] > \alpha[\beta v(1+\theta)+\delta-c]$, we have $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2} > 0$ and $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=0} < 0$. Considering $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=\infty} < 0$, we have $q_1^o(p_1) = 0$.
- $\alpha[\beta v(1+\theta)+\delta-c] \leq p_1 \leq c + \alpha[\beta v(1+\theta) - c]$: We have $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2} \leq 0$ and $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=0} \leq 0$. Again, we have $q_1^o(p_1) = 0$.
- $p_1 > c + \alpha[\beta v(1+\theta) - c]$: We have $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2} < 0$ and $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=0} > 0$. Considering $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\Big|_{q_1=\infty} < 0$, we know that $q_1^o(p_1)$ is the unique solution for $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = 0$, that is, $F(q_1^o) = \frac{p_1 - c - \alpha(\beta v(1+\theta)-c)}{p_1 - \alpha[\beta v(1+\theta)-c+\delta]}$. Together, these three cases prove i). ii) For $p_1 = c + \alpha[\beta v(1+\theta) - c]$, we have $F^{-1}(\frac{p_1 - c - \alpha(\beta v(1+\theta)-c)}{p_1 - \alpha[\beta v(1+\theta)-c+\delta]}) = F^{-1}(0) = 0$, so $q_1^o(p_1)$ is continuous in $p_1 \geq 0$. Since $\frac{p_1 - c - \alpha(\beta v(1+\theta)-c)}{p_1 - \alpha[\beta v(1+\theta)-c+\delta]}$ increases in $p_1 > c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1)$ is non-decreasing in $p_1 \geq 0$. ■

In this chapter, we use superscripts D, h and D, l on order quantity q_1 and/or price p_1 to denote "Dual rollover high price $p_1 = v$" and "Dual rollover low price $p_1 = R_f$", respectively.

Proof of Proposition 2.3.2

We denote the stocking level defined in Lemma 6.1.1 as $q_1^{D,h}(\phi)$ to emphasize its dependence on ϕ. From Lemma 6.1.2, with $p_1 = R_f$, the price and stocking level are independent of ϕ, and thus are denoted by $(p_1^{D,l}, q_1^{D,l})$. According to (6.6) and (6.3),

$$\begin{aligned}\mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})] &= \mathbf{E}[p_1^{D,l}\min\{N, q_1^{D,l}\} - cq_1^{D,l}] + \alpha\int_0^{q_1^{D,l}} \delta(q_1^{D,l} - x)f(x)dx \\ &\quad + \alpha\int_{q_1^{D,l}}^{\infty}[\beta v(1+\theta) - c](x - q_1^{D,l})f(x)dx. \end{aligned} \tag{6.9}$$

$$
\begin{aligned}
&\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)] \\
&= \mathbf{E}[v \min\{(1-\phi)N, q_1^{D,h}(\phi)\} - c q_1^{D,h}(\phi)] \\
&+ \alpha \int_0^{\frac{\delta}{\beta v \phi + \delta(1-\phi)} q_1^{D,h}(\phi)} \{[\beta v(1+\theta) - c](x - q_1^{D,h}(\phi))^+ \\
&+ \delta[q_1^{D,h}(\phi) - \min\{(1-\phi)x, q_1^{D,h}(\phi)\}]\} f(x)dx \\
&+ \alpha \int_{\frac{\delta}{\beta v \phi + \delta(1-\phi)} q_1^{D,h}(\phi)}^{\infty} \{[\beta v(1+\theta) - c](x - q_1^{D,h}(\phi))^+ \\
&+ \beta v[\min\{x, q_1^{D,h}(\phi)\} - \min\{(1-\phi)x, q_1^{D,h}(\phi)\}]\} f(x)dx,
\end{aligned}
$$

Then,

$$
\begin{aligned}
\lim_{\phi \to 0} \mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)] = \mathbf{E}[v \min\{N, q_1^{D,h}(0)\} - c q_1^{D,h}(0)] + \alpha \int_0^{q_1^{D,h}(0)} \delta[q_1^{D,h}(0) - x] f(x)dx \\
+ \alpha \int_{q_1^{D,h}(0)}^{\infty} [\beta v(1+\theta) - c][x - q_1^{D,h}(0)] f(x)dx. \qquad (6.10)
\end{aligned}
$$

We have $\lim_{\phi \to 0} \mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)] \geq \lim_{\phi \to 0} \mathbf{E}[\Pi^{D,h}(q_1^{D,l}, v; \phi)] \geq \mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]$, where the last inequality is from $v \geq p_1^{D,l}$. Here we can see that the high price is optimal when $\phi = 0$.

From Lemma 6.1.1, we know that when $q_1^{D,h}(\phi) > 0$, it solves $\frac{\partial \mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1} = 0$. According to the Envelope Theorem (Mas-Colell et al. 1995), for $q_1^{D,h}(\phi) > 0$, we have

$$
\begin{aligned}
& \frac{d\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)]}{d\phi} \\
= \;& \left. \frac{\partial \mathbf{E}[\Pi^{D,h}(q_1, v; \phi)]}{\partial \phi} \right|_{q_1 = q_1^{D,h}(\phi)} \\
= \;& -v \int_0^{\frac{1}{1-\phi} q_1^{D,h}(\phi)} x f(x)dx + \alpha \int_0^{\frac{\delta}{\beta v \phi + \delta(1-\phi)} q_1^{D,h}(\phi)} \delta x f(x)dx \\
& + \alpha \int_{\frac{\delta}{\beta v \phi + \delta(1-\phi)} q_1^{D,h}(\phi)}^{q_1^{D,h}(\phi)} \beta v x f(x)dx + \alpha \int_{q_1^{D,h}(\phi)}^{\frac{1}{1-\phi} q_1^{D,h}(\phi)} \beta v x f(x)dx + 0
\end{aligned}
$$

$$= \quad (\alpha\delta - v)\int_0^{\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1^{D,h}(\phi)} xf(x)dx + (\alpha\beta v - v)\int_{\frac{\delta}{\beta v\phi+\delta(1-\phi)}q_1^{D,h}(\phi)}^{\frac{1}{1-\phi}q_1^{D,h}(\phi)} xf(x)dx$$
$$< \quad 0.$$

The last inequality is because $\alpha\delta - v < 0$ and $\alpha\beta v - v < 0$. On the other hand, for $q_1^{D,h}(\phi) = 0$, we know $\frac{d\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\,\phi)]}{d\phi} = 0$. Therefore, $\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)]$ is non-increasing in ϕ. Notice that $\mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]$ is constant in ϕ. Let $\phi^{L,D} = \inf\{\phi : \mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)] \leq \mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]$, where $0 \leq \phi < 1\}$. Then, $\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)] \geq \mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]$, if and only if $\phi \leq \phi^{L,D}$. ■

Proof of Proposition 2.3.3

With sales s and market size n, the firm orders $q_2 = n - s$ to sell V2 to every P1NB. The firm receives the revenue σ from each unit of the leftover V1. So, $\Pi_2^S = [\beta v(1+\theta) - c](n - s) + \sigma(q_1 - s)$. ■

Proof of Proposition 2.3.4

The firm can successfully induce all high-end customers to buy in period 1 by setting $p_1 = v$. Therefore, the firm finds the stocking level q_1 by maximizing

$$\mathbf{E}[\Pi^S(q_1, v)] = \mathbf{E}[v\min\{N, q_1\} - cq_1] + \alpha\int_0^\infty [\beta v(1+\theta) - c](x - \min\{x, q_1\})f(x)dx + \alpha\sigma[q_1 - \min\{N, q_1\}], \quad (6.11)$$

with respect to q_1. Note that $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1} = [v - \alpha(\beta v(1+\theta) - c + \sigma)]\overline{F}(q_1) + \alpha\sigma - c$, $\frac{\partial^2\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1^2} = -[v - \alpha(\beta v(1+\theta) - c + \sigma)]f(q_1)$, $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\Big|_{q_1=0} = v - c - \alpha[\beta v(1+\theta) - c]$, and $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\Big|_{q_1=\infty} = \alpha\sigma - c < 0$.

If $v - c - \alpha[\beta v(1+\theta) - c] > 0$, then $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\Big|_{q_1=0} > 0$. Because $v - \alpha[\beta v(1+\theta) - c + \sigma] > v - c - \alpha[\beta v(1+\theta) - c] > 0$, $\mathbf{E}[\Pi^S(q_1, v)]$ is concave in q_1. Therefore, we obtain the optimal solution by solving $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1} = 0$, which results in $q_1^* = F^{-1}(\frac{v-c-\alpha(\beta v(1+\theta)-c)}{v-\alpha[\beta v(1+\theta)-c+\sigma]})$.

If $v - c - \alpha[\beta v(1+\theta) - c] \leq 0$, then $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\Big|_{q_1=0} \leq 0$. Since $\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\Big|_{q_1=\infty} < 0$, no matter whether $\mathbf{E}[\Pi^S(q_1, v)]$ is concave or convex in q_1, we get $q_1^* = 0$. ■

Proof of Proposition 2.3.5

The proof for the low innovation case is below. Since we can prove the results for the medium innovation case in a similar manner, we omit its proof for brevity. Claim i) is the direct result from Lemmas 6.1.1-6.1.2 and Proposition 2.3.4. With $q_1^* = 0$, there is no leftover V1. Therefore, there is no difference between single rollover and dual rollover. This is why we have claim i).

Next, we prove claim ii). Let $\mathbf{E}[\Pi^{S*}]$ be the firm's REE profit under single rollover and q_1^{S*} be the stocking level defined in Proposition 2.3.4. Let $\mathbf{E}[\Pi^{D,h}]$ and $\mathbf{E}[\Pi^{D,l}]$ be the profits associated with solutions defined in Lemma 6.1.1 and Lemma 6.1.2, respectively. Let $\mathbf{E}[\Pi^{D*}] = \max\{\mathbf{E}[\Pi^{D,h}], \mathbf{E}[\Pi^{D,l}]\}$. Therefore, $\mathbf{E}[\Pi^{D*}]$ is the firm's REE profit under dual rollover. From (6.9), (6.10) and (6.11), we have

$$\begin{aligned}\mathbf{E}[\Pi^{D,l}] &= \mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]\\ &= \mathbf{E}[p_1^{D,l}\min\{N, q_1^{D,l}\} - cq_1^{D,l}]\\ &\quad + \alpha[\beta v(1+\theta) - c]\int_{q_1^{D,l}}^{\infty}(x - q_1^{D,l})f(x)dx + \alpha\delta\int_0^{q_1^{D,l}}(q_1^{D,l} - x)f(x)dx,\end{aligned}$$

$$\begin{aligned}\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi] &= \lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v;\phi)]\\ &= \mathbf{E}[v\min\{N, q_1^{D,h}(0)\} - cq_1^{D,h}(0)]\\ &\quad + \alpha[\beta v(1+\theta) - c]\int_{q_1^{D,h}(0)}^{\infty}[x - q_1^{D,h}(0)]f(x)dx\\ &\quad + \alpha\delta\int_0^{q_1^{D,h}(0)}[q_1^{D,h}(0) - x]f(x)dx,\end{aligned}$$

$$\begin{aligned}\mathbf{E}[\Pi^{S*}] &= \mathbf{E}[\Pi^S(q_1^{S*}, v)]\\ &= \mathbf{E}[v\min\{N, q_1^{S*}\} - cq_1^{S*}]\\ &\quad +\alpha[\beta v(1+\theta) - c]\int_{q_1^{S*}}^{\infty}(x - q_1^{S*})f(x)dx + \alpha\sigma\int_0^{q_1^{S*}}(q_1^{S*} - x)f(x)dx,\end{aligned}$$

From Lemmas 6.1.1-6.1.2 and Proposition 2.3.4, we have $q_1 > 0$ when $c+\alpha[\beta v(1+\theta)-c] < v$. In addition, according to the proof of Proposition 2.3.2, with $q_1 > 0$, $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{D,h}]$ decreases in ϕ when $\phi \leq \phi^{L,D}$, and $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{D,l}]$ is constant in ϕ when $\phi > \phi^{L,D}$. Clearly, $\mathbf{E}[\Pi^{S*}]$ is also constant in ϕ. By comparing $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi]$ and $\mathbf{E}[\Pi^{S*}]$ above, we have $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi] = \lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v;\phi)] \geq \lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{S*}, v;\phi)] \geq \mathbf{E}[\Pi^{S}(q_1^{S*}, v)] = \mathbf{E}[\Pi^{S*}]$, where the last inequality is due to $\delta > \sigma$. Note that in the extreme case $\sigma = \delta$, $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1, v;\phi)] = \mathbf{E}[\Pi^{S}(q_1, v)]$. Thus, in this extreme case, we have $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi] = \mathbf{E}[\Pi^{S*}]$. Because $\mathbf{E}[\Pi^{S*}]$ is increasing in σ, if we decrease σ by starting with $\sigma = \delta$, there exists a threshold $\triangle \geq 0$ such that for $\delta - \sigma \leq \triangle$, there exists a ϕ^L, $\phi^L < \phi^{L,D}$, at which $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{D,h}] = \mathbf{E}[\Pi^{S*}]$. In addition, single rollover is optimal iff $\phi \geq \phi^L$. This proves claim ii.a). When $\delta - \sigma > \triangle$, no ϕ can set $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{S*}]$. In this case, dual rollover is optimal. This proves claim ii.b). ■

Proof of Proposition 2.4.1

With high innovation, the firm can profitably target P1B with V2 by setting $p_2 = \beta v\theta$. This increases the number of targeting strategies, from 3 in the low innovation cases, to 7: [{B}, {P1NB}], [{B}, {P1B,P1NB}], [{P1NB}, {P1NB}], [{P1NB}, {P1B,P1NB}], [{B,P1NB}, {P1NB}], [{B,P1NB}, {P1B}], [{B,P1NB}, {P1B,P1NB}]. Based on our analysis, we find that the strategies used in period 2 are L-L, L-M and H-H as listed in Proposition 2.4.1, which result in [{B}, {P1B,P1NB}], [{B}, {P1NB}] and [{P1NB}, {P1NB}]. Similar to the proof of Proposition 2.3.1, we investigate the conditions under which a certain strategy is optimal and compute the expected waiting surplus. For brevity, we omit the details. ■

Proof of Proposition 2.4.2

With $p_1 = R_f$, we have $s = \min\{n, q_1\}$. The firm's profit is

$$\begin{aligned}\mathbf{E}[\Pi^{D,l}(q_1, R_f)] &= \mathbf{E}[R_f \min\{N, q_1\} - cq_1] \\ &\quad + \alpha\int_0^{\frac{\beta v(1+\theta)-c}{\beta v}q_1} [(\beta v\theta - c)x + \delta(q_1 - \min\{x, q_1\})]f(x)dx\end{aligned}$$

$$+ \alpha \int_{\frac{\beta v(1+\theta)-c}{\beta v}q_1}^{\infty} [(\beta v(1+\theta)-c)(x-\min\{x,q_1\})] f(x)dx. \tag{6.12}$$

We have $R_f = v - \beta v F\left(\frac{\beta v(1+\theta)-c}{\beta v} q_1\right)$ from (2.7), (2.8) and Proposition 2.4.1. Similar to (6.7)-(6.8), we have (6.13)-(6.14) below. We need to investigate if and how many times $q_1(p_1)$ and $p_1(q_1)$ cross each other in three steps. *Step 1:* For (6.13), how the optimal q_1 changes as p_1 increases; *Step 2:* For (6.14), how p_1 changes as q_1 increases; and *Step 3:* Under which conditions (6.13) and (6.14) have crossing point(s).

$$q_1(p_1) = \arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1,p_1)], \tag{6.13}$$

$$p_1(q_1) = v(1 - \beta F(\frac{\beta v(1+\theta)-c}{\beta v} q_1)). \tag{6.14}$$

Step 1:

$$\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = p_1 \overline{F}(q_1) - c - \alpha[\beta v(1+\theta)-c]\overline{F}(\frac{\beta v(1+\theta)-c}{\beta v} q_1) + \alpha\delta F(q_1),$$

$$\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} = p_1 - c - \alpha[\beta v(1+\theta)-c],$$

$$\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=\infty} = \alpha\delta - c < 0,$$

and

$$\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2} = f(q_1)\left[\alpha\delta - p_1 + \frac{\alpha[\beta v(1+\theta)-c]^2}{\beta v}\frac{f([1+\theta-c/(\beta v)]q_1)}{f(q_1)}\right].$$

Since $f(\cdot)$ has the MSLR property, if $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2}$ crosses 0, then it crosses at most once (cross-once property). We also know $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial p_1 \partial q_1} = \overline{F}(q_1) > 0$.

From $\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} = p_1 - c - \alpha[\beta v(1+\theta)-c]$, the shape of $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ around $q_1 = 0$ depends on p_1. Particularly, $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]|_{q_1=0} < 0$ if and only if $p_1 < c + \alpha[\beta v(1+\theta)-c]$. Therefore, $\arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ depends on the value of p_1. Next we investigate $\arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ in three cases described by increasing p_1:

Case I): $p_1 = 0$.

Case II): $0 < p_1 \le c + \alpha[\beta v(1+\theta) - c]$. This case has two subcases:

Case II.a): $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] < 0$ for all $q_1 > 0$ and all $p_1 \in (0, c + \alpha[\beta v(1+\theta) - c])$.

Case II.b): $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] \ge 0$ for some $q_1 > 0$ and some $p_1 \in (0, c + \alpha[\beta v(1+\theta) - c])$.

Case III): $p_1 > c + \alpha[\beta v(1+\theta) - c]$.

We analyze how the optimal q_1 changes as p_1 increases in all these cases in Lemmas 6.1.4 and 6.1.5, which are stated below but proved later.

Lemma 6.1.4 *Under case II.b),*

i) As p_1 increases, the number of roots of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ increases from zero to one, and reaches finally two.

ii) If there is no or one root for $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$, then the optimal stocking level is zero.

iii) If there are two roots for $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$, then

$$\mathbf{E}[\Pi^{D,l}(q_1^-(p_1), p_1)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1), p_1)],$$

where $q_1^-(p_1)$ and $q_1^+(p_1)$ are the smaller and larger roots, respectively. Moreover, the smaller root $q_1^-(p_1)$ decreases in p_1 and the larger root $q_1^+(p_1)$ increases in p_1.

iv) If $p_1 = c + \alpha[\beta v(1+\theta) - c]$, then $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ first increases and then decreases. Moreover, $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ has a unique positive maximizer $q_1^+(p_1)$, which is the larger root for $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$.

v) If there are two roots of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$, then there exists a critical unique price p_1^J such that $\mathbf{E}[\Pi^{D,l}(0, p_1^J)] = \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J), p_1^J)]$ and $p_1^J < c + \alpha[\beta v(1+\theta) - c]$. In addition, $\mathbf{E}[\Pi^{D,l}(q_1^+(p_1), p_1)] >$ (resp., $<$) $\mathbf{E}[\Pi^{D,l}(0, p_1)]$ when $p_1 >$ (resp., $<$) p_1^J.

Lemma 6.1.5 *For a given* p_1*, there exists a maximizer* $q_1^o(p_1) \geq 0$ *of* $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$. $q_1^o(p_1)$ *satisfies:*

i) In cases I), II.a), and III),

$$q_1^o(p_1) = \begin{cases} 0 & \text{if case I for } p_1 = 0; \\ 0 & \text{if case II.a) for } 0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c], \\ \text{the unique positive root of } \frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = 0 & \text{if case III for } p_1 > c + \alpha[\beta v(1+\theta) - c]. \end{cases}$$

ii) In case II.b) for $0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c]$,

$$q_1^o(p_1) = \begin{cases} 0 & \text{if } 0 < p_1 \leq p_1^J, \\ q_1^+(p_1) & \text{if } p_1^J \leq p_1 \leq c + \alpha[\beta v(1+\theta) - c], \end{cases}$$

where p_1^J *and* $q_1^+(p_1)$ *are defined in Lemma 6.1.4.*

iii) In all cases, $q_1^o(p_1)$ *is non-decreasing in* p_1 *for all* $p_1 \geq 0$.

iv) If case II.a) occurs when $0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c]$*, then combining cases I), II.a), and III),* $q_1^o(p_1)$ *is continuous in* p_1 *for all* $p_1 \geq 0$*. If cases II.b) occurs when* $0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c]$*, then combining cases I), II.b), and III),* $q_1^o(p_1)$ *is continuous everywhere except at* $p_1 = p_1^J$.

Step 2: Solving q_1 from (6.14), we have $q_1^{-1}(p_1) = (\frac{\beta v}{\beta v(1+\theta) - c})F^{-1}(\frac{1}{\beta}(1 - \frac{p_1}{v}))$. Clearly, q_1 decreases in p_1. In addition, we know that $q_1 = 0$ when $p_1 = v$ and $q_1 = \infty$ when $p_1 = v(1-\beta)$; and it is impossible that $p_1 > v$ or $p_1 < v(1-\beta)$. For ease of exposition, we define the $q_1^{-1}(p_1)$ as

$$q_1^{-1}(p_1) = \begin{cases} (\frac{\beta v}{\beta v(1+\theta) - c})F^{-1}(\frac{1}{\beta}(1 - \frac{p_1}{v})) & \text{if } \ v(1-\beta) \leq p_1 \leq v, \\ \infty & \text{if } \ p_1 < v(1-\beta). \end{cases}$$

This extended function $q_1^{-1}(p_1)$ does not change the existence or not of the crossing point.

Step 3: Steps 1-2 show that if there is a crossing point between (6.13) and (6.14), then the crossing point must be unique due to the non-decreasing property of $q_1^o(p_1)$ in (6.13) and the decreasing property of $q_1^{-1}(p_1)$ in (6.14). In addition, a crossing point corresponds to a unique vector $(q_1, p_1, \chi, W_c, R_f)$ satisfying the REE conditions except (2.11). Therefore, to show the existence of the vector $(q_1, p_1, \chi, W_c, R_f)$, we just need to show that there is a crossing point between (6.13) and (6.14). We denote the crossing point (if any) as (p_1, q_1). Now let us study the conditions under which (6.13) and (6.14) have the crossing point.

If case II.a) occurs, then $q_1^o(p_1)$ is continuous and non-decreasing in p_1 for all p_1's, and there must be a crossing point.

If case II.b) occurs, then a crossing point does not exist if and only if all of the following conditions are satisfied: $v(1-\beta) < p_1^J < v$, and $q_1^+(p_1^J) > (\frac{\beta v}{\beta v(1+\theta)-c})F^{-1}(\frac{1}{\beta}(1 - \frac{p_1^J}{v}))$. Under these conditions, if we can show that there exists a unique a combination of $(q_1^-, p_1, \chi, W_c, R_f)$ and $(q_1^+, p_1, \chi, W_c, R_f)$ satisfying the REE conditions except (2.11), then we complete the proof of Proposition 2.4.2. According to Lemma 6.1.4, at the jump point $p_1 = p_1^J$, $\mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J), p_1^J)] = \mathbf{E}[\Pi^{D,l}(0, p_1^J)] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$, which means both $q_1 = q_1^+(p_1^J)$ and $q_1 = 0$ maximize $\mathbf{E}[\Pi^{D,l}(q_1, p_1^J)]$. Therefore, if the firm orders $q_1 = 0$ with probability λ $(0 < \lambda < 1)$ and $q_1 = q_1^+(p_1^J)$ with probability $1-\lambda$, then it can still get the maximum profit. The resulting expected waiting surplus is $\lambda\beta v F(\frac{\beta v(1+\theta)-c}{\beta v} \times 0) + (1-\lambda)\beta v F(\frac{\beta v(1+\theta)-c}{\beta v} q_1^+(p_1^J)) = (1-\lambda)\beta v F(\frac{\beta v(1+\theta)-c}{\beta v} q_1^+(p_1^J))$, and thus we must have $\chi = 1$, $W_c = (1-\lambda)\beta v F(\frac{\beta v(1+\theta)-c}{\beta v} q_1^+(p_1^J))$ and $p_1^J = R_f = v - W_c$. This means we need to show that there exists a λ satisfying $p_1^J = v(1 - \beta(1-\lambda)F(\frac{\beta v(1+\theta)-c}{\beta v} q_1^+(p_1^J)))$. Since $q_1^+(p_1^J) > (\frac{\beta v}{\beta v(1+\theta)-c})F^{-1}(\frac{1}{\beta}(1 - \frac{p_1^J}{v}))$, we have $p_1^J > v(1 - \beta F(\frac{\beta v(1+\theta)-c}{\beta v} q_1^+(p_1^J)))$. Moreover, we know $p_1^J < v$, so we can always find a $\lambda \in (0,1)$ satisfying $p_1^J = v(1-\beta(1-\lambda)F(\frac{\beta v(1+\theta)-c}{\beta v} q_1^+(p_1^J)))$. Letting $q_1^+ = q_1^+(p_1^J)$ and $q_1^- = 0$, we complete the proof. ■

Proof of Lemma 6.1.4

i) When $p_1 = 0$, $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] < 0$ for all q_1. From $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial p_1 \partial q_1} > 0$, we know that

$[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]$ increases as p_1 increases. From the definition of case II.b) we know that $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]\ /\partial q_1] \geq 0$ for some $q_1 > 0$ and some $p_1 \in (0, c+\alpha[\beta v(1+\theta)-c])$. Because $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]$ is continuous in p_1, if we increase p_1 by starting with $p_1 = 0$, then we can always find a $\widehat{p}_1 \in (0, c+\alpha[\beta v(1+\theta)-c])$ such that $[\partial \mathbf{E}[\Pi^{D,l}(q_1,\widehat{p}_1)]/\partial q_1] = 0$ for some $q_1 > 0$ and $[\partial \mathbf{E}[\Pi^{D,l}(q_1,\widehat{p}_1)]/\partial q_1] < 0$ for other q_1's. Because $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] < 0$ for all $q_1 > 0$ when $p_1 < \widehat{p}_1$, and $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] > 0$ for some $q_1 > 0$ when $p_1 > \widehat{p}_1$, we know that $\widehat{p}_1$ is unique. Next, we prove by contradiction that when $p_1 = \widehat{p}_1$, there is only one $q_1 > 0$ at which $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. Suppose that there are two q_1's at which $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. Because $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] < 0$ for other q_1's, the sign of $[\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1^2]$ changes more than once, which contradicts with the cross-once property. Similarly, we can prove that it is impossible to have more than two q_1's at which $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. Therefore, there is only one root for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$ when $p_1 = \widehat{p}_1$, and there is no root for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$ when $p_1 < \widehat{p}_1$.

Now we prove that when $p_1 > \widehat{p}_1$, there are two roots for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. From analysis above we know that when $p_1 > \widehat{p}_1$, $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] > 0$ for some $q_1 > 0$. Because $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]|_{q_1=0} < 0$ and $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]|_{q_1=\infty} < 0$, we know that there are at least two roots for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. If there are more than two roots, then the sign of $[\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1^2]$ must change more than once, which contradicts with the cross-once property. Therefore, there are two roots for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$ when $p_1 > \widehat{p}_1$. Combining the analysis above, we get claim i).

ii) Because $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] < 0$ for all $q_1 > 0$ when $p_1 < \widehat{p}_1$, the optimal stocking level is zero. Similarly, because when $p_1 = \widehat{p}_1$, $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$ for only one q_1 and $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] < 0$ for other q_1's, we know that the optimal stocking level is zero.

iii) When $p_1 > \widehat{p}_1$, there are two roots for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. Together with $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]\ /\partial q_1]|_{q_1=0} < 0$ and $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]|_{q_1=\infty} < 0$, we know that as q_1 increases, $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ is decreasing for $q_1 \leq q_1^-(p_1)$, increases for $q_1^-(p_1) \leq q_1 \leq q_1^+(p_1)$, and decreases for $q_1 \geq q_1^+(p_1)$. This implies $\mathbf{E}[\Pi^{D,l}(q_1^-(p_1),p_1)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$.

For $p_1 > \widehat{p}_1$, $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]$ is positive only for $q_1 \in [q_1^-(p_1), q_1^+(p_1)]$. Since $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]$ increases in p_1 for all q_1's, $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1+\varepsilon)]/\partial q_1]$ is positive over a larger interval for $q_1 \in [q_1^-(p_1+\varepsilon), q_1^+(p_1+\varepsilon)]$ that includes the original interval. Thus, $q_1^-(p_1+\varepsilon) \le q_1^-(p_1)$ and $q_1^+(p_1+\varepsilon) \ge q_1^+(p_1)$.

iv) When $p_1 = c + \alpha[\beta v(1+\theta) - c]$, we have $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]|_{q_1=0} = 0$. With $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]|_{q_1=\infty} < 0$ and the cross-once property, the only two possibilities for $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ is that (a) $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] < 0$ for all $q_1 > 0$, and (b) $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ first increases, reaches a root, and then decreases. As $\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial p_1 \partial q_1} > 0$, $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1]$ increases as p_1 increases. So (a) implies that $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] < 0$ for all $q_1 > 0$ for all $p_1 < c + \alpha[\beta v(1+\theta) - c]$, which violates the definition of case II.b). So (b) is the only possible case. Further, $q_1^-(p_1) = 0$, and $q_1^+(p_1)$ is the unique positive maximizer. Claim iv) follows.

v) We first show that there is a unique p_1^J. Note that (a) $\mathbf{E}[\Pi^{D,l}(0,p_1)] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$ regardless of the value of p_1, (b) $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ is continuous and increasing in p_1 for each fixed $q_1 > 0$, (c) $\mathbf{E}[\Pi^{D,l}(q_1,\widehat{p}_1)] < \mathbf{E}[\Pi^{D,l}(0,\widehat{p}_1)]$ for all q_1's, and (d) when $p_1 > \widehat{p}_1$, there are two roots for $[\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1] = 0$. Starting from $p_1 = \widehat{p}_1$ and increasing p_1, we can always find a unique p_1^J at which $\mathbf{E}[\Pi^{D,l}(0,p_1^J)] = \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)]$. Further, if $p_1 < p_1^J$, then $\mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1^J)] \le \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)] = \mathbf{E}[\Pi^{D,l}(0,p_1^J)]$. If $p_1 > p_1^J$, then $\mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)] \ge \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1)] > \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)] = \mathbf{E}[\Pi^{D,l}(0,p_1^J)]$.

Next we show that $p_1^J < c + \alpha[\beta v(1+\theta) - c]$ by contradiction. If $p_1^J = c + \alpha[\beta v(1+\theta) - c]$, we know $\mathbf{E}[\Pi^{D,l}(0,p_1^J)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)]$ from iv), which violates the definition of p_1^J. Because $\mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$ increases in p_1 and $\mathbf{E}[\Pi^{D,l}(0,p_1)]$ is constant in p_1, if $p_1^J > c + \alpha[\beta v(1+\theta) - c]$, then we again have $\mathbf{E}[\Pi^{D,l}(0,p_1^J)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)]$. Therefore, $p_1^J < c + \alpha[\beta v(1+\theta) - c]$. ■

Proof of Lemma 6.1.5

i) Case I): When $p_1 = 0$, $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = -c - \alpha[\beta v(1+\theta) - c]\overline{F}(\frac{\beta v(1+\theta)-c}{\beta v} q_1) + \alpha\sigma F(q_1) < 0$ all q_1. This means $q_1 = 0$ is the optimal solution.

Case II.a): When $0 < p_1 < c + \alpha[\beta v(1+\theta) - c]$, $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} < 0$ for all $q_1 \geq 0$. So $q_1 = 0$ maximizes $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$. By a limit argument, when $p_1 = c + \alpha[\beta v(1+\theta) - c]$, we have $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} \leq 0$ for all $q_1 \geq 0$. $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ cannot be constant over an interval of q_1, otherwise $[\partial^2 \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1^2]$ is zero over the interval, which violates the cross-once property. Thus, $q_1^o(p_1) = 0$ when $p_1 = c + \alpha[\beta v(1+\theta) - c]$.

Case III): When $p_1 > c + \alpha[\beta v(1+\theta) - c]$, by combining $\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} > 0$, $\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0$ and the cross-once property, we know that $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ is unimodal. In addition, there exists a unique positive root for $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = 0$, and this unique positive root maximizes $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$.

ii) According to Lemma 6.1.4, when $0 < p_1 \leq p_1^J$, $q_1^o(p_1) = 0$ for the scenarios in which there is zero or one root, or there are two roots but $\mathbf{E}[\Pi^{D,l}(0, p_1)] \geq \mathbf{E}[\Pi^{D,l}(q_1^+(p_1), p_1)]$. When $p_1^J \leq p_1 \leq c+\alpha[\beta v(1+\theta)-c]$, there are two roots and $\mathbf{E}[\Pi^{D,l}(0, p_1)] \leq \mathbf{E}[\Pi^{D,l}(q_1^+(p_1), p_1)]$.

iii) For cases I) and II.a), $q_1^o(p_1)$ is constant in p_1. For case II.b), we have increasing $q_1^+(p_1)$ from Lemma 6.1.4.iii). We can prove the increasing property of $q_1^o(p_1)$ for case III) in the same way as the proof of Lemma 6.1.4.iii) except that there is a single root here. This single root satisfies the properties that $q_1^+(p_1)$ satisfies in Lemma 6.1.4.iii).

iv) Cases I), II.a), and III): The unique root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ is continuous in p_1 in case III). From the analysis for case II.a) above, we know that when $p_1 = c+\alpha[\beta v(1+\theta)-c]$, $q_1 = 0$ is maximizer for $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ as well as the unique root for $\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = 0$. Therefore, $q_1^o(p_1)$ is continuous in p_1 for all $p_1 \geq 0$ when combining cases I), II.a), and III).

Cases I), II.b), and III): Notice that when $p_1 > c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1)$ is the unique root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ and thus is continuos. When $p_1^J \leq p_1 \leq c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1) = q_1^+(p_1)$ is always the larger root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ and is also continuous. Because there is a unique positive root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ when $p_1 > c + \alpha[\beta v(1+\theta) - c]$, the unique positive root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ is corresponding to $q_1^+(p_1)$ rather than $q_1^-(p_1)$ for $p_1^J \leq p_1 \leq c + \alpha[\beta v(1+\theta) - c]$. That is, the unique positive root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ when $p_1 > c + \alpha[\beta v(1+\theta) - c]$

evolves from $q_1^+(p_1)$ for $p_1^J \leq p_1 \leq c + \alpha[\beta v(1+\theta) - c]$. Otherwise, we would need another root corresponding to $q_1^+(p_1)$, which violates the fact that there is a unique positive root of $[\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1] = 0$ when $p_1 > c + \alpha[\beta v(1+\theta) - c]$. So, $q_1^o(p_1)$ is continuous at $p_1 = c + \alpha[\beta v(1+\theta) - c]$. Finally, $q_1^o(p_1)$ jumps from 0 to $q_1^+(p_1^J)$ only at $p_1 = p_1^J$. ■

Proof of Proposition 2.4.3

We have argued for the strategies L and H in the main body. Similar to the proof of Proposition 2.3.1, we can investigate the conditions under which a certain strategy is optimal. In case $[p_1 = R_f]$, following (2.12) and (2.15), we have $w(q_1, R_f, 1) = \int_0^{\frac{\beta v(1+\theta)-c}{\beta v} q_1} 1 \times [\beta v(1+\theta) - \beta v\theta] f(x)dx + 0 = \beta v F(\frac{\beta v(1+\theta)-c}{\beta v} q_1)$. In case $[p_1 = v]$, if $\phi \leq \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, then we can compute $w(q_1, v, 0)$ in the same way as in $[p_1 = R_f]$; if $\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, then $w(q_1, v, 0) = 0$ since the firm always uses H strategy. ■

Lemma 6.1.6 (High innovation) *With $\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, there is no REE under single rollover where $p_1^* = v$ and $\chi = 0$.*

Proof. If there is an REE with $p_1^* = v$ and $\chi = 0$, then we have $R_f < v$. Otherwise, if $R_f = v$, then $\chi = 1$. However, with $p_1 = v$ and $\chi = 0$, we have $w(q_1, v, 0) = 0$ from Proposition 2.4.3, which leads to $R_f = v$, a contradiction with $R_f < v$. Thus, such an REE does not exist. ■

Proving Proposition 2.4.4 requires two results which are stated and proved below as Lemmas 6.1.7 and 6.1.8. Lemmas 6.1.7 and 6.1.8 are analogous to Proposition 2.3.2 but for the case with high innovation under dual rollover and the case with high innovation under single rollover, respectively.

Lemma 6.1.7 (High innovation) *Under dual rollover, there exists a unique REE. In addition, a $\phi^{H,D}$ exists such that if $\phi \leq \phi^{H,D}$, then the firm sets the high price $p_1^* = v$; otherwise, the firm sets the low price $p_1^* = R_f$.*

Proof. From Proposition 2.4.1, with $p_1 = v$, the strategies in period 2 depend on ϕ and can be divided into three cases; while with $p_1 = R_f$, the strategies hold for all ϕs. Define

$\mathbf{E}[\Pi^{D,h-All}(q_1^{D,h}(\phi), v; \phi)]$ as the firm's total profit with $p_1 = v$ across these three cases. After writing the profits for these three cases, it is easy to show that $\mathbf{E}[\Pi^{D,h-All}(q_1^{D,h}(\phi), v; \phi)]$ is continuous in ϕ by checking the limits of the integrals and the integrands at $\phi = \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$ and $\phi = \frac{\beta v\theta - c}{\beta v\theta + \delta - c}$. As in Proposition 2.3.2, we can show that $\mathbf{E}[\Pi^{D,h-All}(q_1^{D,h}(\phi), v; \phi)]$ is non-increasing in ϕ by using the Envelope Theorem.

If a unique $(q_1^{D,l}, p_1^{D,l}, \chi, W_c, R_f)$ satisfying the REE conditions except (2.11) exists, then $\mathbf{E}[\Pi^{D,l}(p_1^{D,l}, q_1^{D,l})]$ in (6.12) is constant in ϕ. If the vector of numbers $(q_1^{D,l}, p_1^{D,l}, \chi, W_c, R_f)$ does not exist, then from Proposition 2.4.2, $\mathbf{E}[\Pi^{D,l}(p_1^{D,l}, q_1^{D,l})] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$ is also constant in ϕ. Defining $\phi^{H,D}$ as $\phi^{H,D} = \inf\{\phi : \mathbf{E}[\Pi^{D,h-All}(q_1^{D,h}(\phi), v; \phi)] \le \mathbf{E}[\Pi^{D,l}(p_1^{D,l}, q_1^{D,l})],$ where $0 \le \phi < 1\}$, we have the desired result. ■

Lemma 6.1.8 (High innovation) *Under single rollover, there exists either a unique REE or a unique zero-order-mixed REE. In addition, there exists a* $\phi^{H,S}$, $0 \le \phi^{H,S} \le \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, *such that if* $\phi \le \phi^{H,S}$, *then the firm sets the high price* $p_1^* = v$; *otherwise, the firm sets the low price* $p_1^* = R_f$.

Proof. The proof of Lemma 6.1.8 is similar to that of Lemma 6.1.7. The difference is that for $\phi > \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, according to Lemma 6.1.6, the only equilibrium is with $p_1^* = R_f$ and $\chi = 1$. So, we define $\phi^{H,S} = \min\{\bar{\phi}, \frac{\beta v\theta - c}{\beta v(1+\theta)-c}\}$, where $\bar{\phi} = \inf\{\phi : \mathbf{E}[\Pi^{S,h}(q_1^{S,h}(\phi), v; \phi)] \le \mathbf{E}[\Pi^{S,l}(q_1^{S,l}, p_1^{S,l})]$, where $0 \le \phi < 1\}$, $\mathbf{E}[\Pi^{S,h}(q_1^{S,h}(\phi), v; \phi)]$ is the profit with $p_1 = v$ for $\phi \le \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$ and $\mathbf{E}[\Pi^{S,l}(q_1^{S,l}, p_1^{S,l})]$ is the profit with $p_1 = R_f$. The desired result follows. ■

Proof of Proposition 2.4.4

i) Note that when $\phi \le \phi^{H,S}$, we have $\mathbf{E}[\Pi^{D*}] \ge \mathbf{E}[\Pi^{D,h}] = \mathbf{E}[\Pi^{D,h}(q_1^{D,h}, v)] > \mathbf{E}[\Pi^{D,h}(q_1^{S,h}, v)]$ $> \mathbf{E}[\Pi^{S,h}(q_1^{S,h}, v)] = \mathbf{E}[\Pi^{S,h}] = \mathbf{E}[\Pi^{S*}]$. We have the last inequality by observing that when $\phi \le \phi^{H,S} \le \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, the expressions of $\mathbf{E}[\Pi^{D,h}(q_1, v)]$ and $\mathbf{E}[\Pi^{S,h}(q_1, v)]$ are structurally the same, and the only difference between them lies in δ v.s. σ, and $\delta > \sigma$. Therefore, $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi \le \phi^{H,S}$. This implies $\phi^H > \phi^{H,S}$.

From the proofs of Lemmas 6.1.7 and 6.1.8, we know that $\mathbf{E}[\Pi^{D*}]$ is continuous in ϕ, and $\mathbf{E}[\Pi^{S*}]$ is continuous in ϕ except that if $\phi^{H,S} = \frac{\beta v\theta - c}{\beta v(1+\theta)-c}$, then there may be a drop at

$\phi = \phi^{H,S}$. In addition, $\mathbf{E}[\Pi^{D*}]$ (resp., $\mathbf{E}[\Pi^{S*}]$) is non-increasing in ϕ when $\phi < \phi^{H,D}$ (resp., $\phi < \phi^{H,S}$), and constant in ϕ when $\phi \geq \phi^{H,D}$ (resp., $\phi \geq \phi^{H,S}$). Since $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi \leq \phi^{H,S}$, if there does not exist a ϕ^H such that $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{D*}]$ at $\phi = \phi^H$, then dual rollover is always better than single rollover. This proves claim i.a).

Next, we prove claim i.b). First, we can prove that if $\phi^{H,S} \geq \phi^{H,D}$, then ϕ^H does not exist. This is because when $\phi^{H,S} \geq \phi^{H,D}$ we have (a) for $\phi < \phi^{H,D} \leq \phi^{H,S}$, $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$, and (b) for $\phi \geq \phi^{H,D}$, $\mathbf{E}[\Pi^{D*}]$ is continuous and constant in ϕ and $\mathbf{E}[\Pi^{S*}]$ is non-increasing in ϕ. Facts (a) and (b) together show that $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ for all ϕ's, and thus ϕ^H does not exist. So, if ϕ^H exists, we must have $\phi^{H,S} < \phi^{H,D}$. In addition, when $\phi > \phi^{H,D} > \phi^{H,S}$, both $\mathbf{E}[\Pi^{S*}]$ and $\mathbf{E}[\Pi^{D*}]$ are continuous and constant. So, if ϕ^H exists, we must have $\phi^H \leq \phi^{H,D}$. Together with the fact $\phi^H > \phi^{H,S}$ from analysis above, we know $\phi^{H,S} < \phi^H \leq \phi^{H,D}$.

When $\phi > \phi^H$, because $\mathbf{E}[\Pi^{S*}]$ is continuous and constant and $\mathbf{E}[\Pi^{D*}]$ is continuous and non-increasing, we have $\mathbf{E}[\Pi^{S*}] \geq \mathbf{E}[\Pi^{D*}]$. When $\phi \leq \phi^{H,S} < \phi^H$, we have $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ from our analysis above. In addition, because $\mathbf{E}[\Pi^{D*}]$ is non-increasing while $\mathbf{E}[\Pi^{S*}]$ is constant when $\phi^{H,S} < \phi < \phi^H$ and $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{S*}]$ when $\phi = \phi^H$, we have again $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$. Therefore, $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi < \phi^H$. Combining the cases of $\phi > \phi^H$ and $\phi < \phi^H$, we get claim i.b).

ii) If there is a zero-order-mixed REE under single rollover, then $\mathbf{E}[\Pi^{S*}] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$. Since $\mathbf{E}[\Pi^{D*}] \geq \mathbf{E}[\Pi^{D,h}] = \mathbf{E}[\Pi^{D,h}(q_1^{D,h}, v)] \geq \mathbf{E}[\Pi^{D,h}(0, v)] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$, dual rollover is always better than single rollover.

Proof of Proposition 2.4.5

Claim i) is from the proof of Proposition 2.4.4: when $\phi \leq \phi^{H,S}$, that is, when $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,h}]$, we have $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$. This shows that single rollover can outperform dual rollover only when $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,l}]$. If $\mathbf{E}[\Pi^{S,l}] < \mathbf{E}[\Pi^{D,l}]$, then $\mathbf{E}[\Pi^{D*}] \geq \mathbf{E}[\Pi^{D,l}] > \mathbf{E}[\Pi^{S,l}]$, which means that ϕ^H does not exist. Thus, $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$ is a necessary condition for the existence of ϕ^H. Next we prove that $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$ is also the sufficient condition for the existence of ϕ^H. Note that $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi \leq \phi^{H,S}$, and $\mathbf{E}[\Pi^{D*}]$ is continuous and constant in ϕ when $\phi \geq \phi^{H,D}$, but $\mathbf{E}[\Pi^{S*}]$ is non-increasing in ϕ and $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,l}]$ when $\phi \geq \phi^{H,S}$. Hence, if $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$, then such a ϕ^H must exist. ■

6.2 Proofs related to Chapter 3

The proofs of Lemma 3.4.1 i) and ii.a) are similar to those of Lemma 2 in Liang et al. (2011a). For ease of reading, we write down all the steps below. Before we prove Lemma 3.4.1, we first need to prove Lemma 6.2.1.

Lemma 6.2.1 (S-DR) *For a given p_1, there exists a maximizer $q_1^o(p_1) \geq 0$ of $\pi^{\text{S-DR}}(q_1, p_1, \theta)$. Furthermore, $q_1^o(p_1)$ satisfies:*

i)

$$q_1^o(p_1) = \begin{cases} 0 & \text{if } 0 < p_1 \leq c + \alpha[v(1+\theta) - c], \\ F^{-1}\left(\frac{p_1 - c - \alpha[v(1+\theta)-c]}{p_1 - \alpha[v(1+\theta)-c] - \alpha\delta}\right) & \text{if } p_1 > c + \alpha[v(1+\theta) - c] \end{cases}$$

ii) $q_1^o(p_1)$ is continuous and non-decreasing in p_1 for all $p_1 \geq 0$.

Proof. From (3.6), we have $\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1} = p_1\overline{F}(q_1) - c - \alpha[v(1+\theta) - c]\overline{F}(q_1) + \alpha\delta F(q_1)$, $\frac{\partial^2 \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1^2} = f(q_1)\{\alpha[v(1+\theta) + \delta - c] - p_1\}$, $\left.\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\right|_{q_1=0} = p_1 - c - \alpha[v(1+\theta) - c]$ and $\left.\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\right|_{q_1=\infty} = \alpha\delta - c < 0$.

i) We investigate the maximizer $q_1^o(p_1)$ of $\pi^{\text{S-DR}}(q_1, p_1, \theta)$ in three cases described by increasing p_1:

- $p_1 < \alpha[v(1+\theta) + \delta - c]$: Since $c + \alpha[v(1+\theta) - c] > \alpha[v(1+\theta) + \delta - c]$, we have $\frac{\partial^2 \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1^2} > 0$ and $\left.\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\right|_{q_1=0} < 0$. Considering $\left.\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\right|_{q_1=\infty} < 0$, we have $q_1^o(p_1) = 0$.
- $\alpha[v(1+\theta) + \delta - c] \leq p_1 \leq c + \alpha[v(1+\theta) - c]$: We have $\frac{\partial^2 \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1^2} \leq 0$ and $\left.\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\right|_{q_1=0} \leq 0$. Again, we have $q_1^o(p_1) = 0$.
- $p_1 > c + \alpha[v(1+\theta) - c]$: We have $\frac{\partial^2 \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1^2} < 0$ and $\left.\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\right|_{q_1=0} > 0$. Considering

$\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1}\Big|_{q_1=\infty} < 0$, we know that $q_1^o(p_1)$ is the unique solution for $\frac{\partial \pi^{\text{S-DR}}(q_1,p_1,\theta)}{\partial q_1} = 0$, that is, $F(q_1^o) = \frac{p_1-c-\alpha[v(1+\theta)-c]}{p_1-\alpha[v(1+\theta)-c]-\alpha\delta}$. Together, these three cases prove i).

ii) For $p_1 = c + \alpha[v(1+\theta) - c]$, we have $F^{-1}(\frac{p_1-c-\alpha[v(1+\theta)-c]}{p_1-\alpha[v(1+\theta)-c]-\alpha\delta}) = F^{-1}(0) = 0$, so $q_1^o(p_1)$ is continuous in $p_1 \geq 0$. Since $\frac{p_1-c-\alpha[v(1+\theta)-c]}{p_1-\alpha[v(1+\theta)-c]-\alpha\delta}$ increases in $p_1 > c + \alpha[v(1+\theta) - c]$, $q_1^o(p_1)$ is non-decreasing in $p_1 \geq 0$. ■

Proof of Lemma 3.4.1

From the rational expectation equilibrium conditions (i), (iii) and (iv), we have $p_1 = R_f = v - (v-\delta)F(q_1)$. Together with condition (ii), if we can obtain unique q_1 and p_1 satisfying the two equations

$$\begin{cases} q_1 &= \arg\max_{q_1} \pi^{\text{S-DR}}(q_1, p_1, \theta), \\ p_1 &= v - (v-\delta)F(q_1), \end{cases} \tag{6.15}$$

then there is a unique equilibrium for any given θ. For any given θ, by treating the right-hand sides in (6.15) as functions of p_1 and q_1, we have

$$q_1(p_1) = \arg\max_{q_1} \pi^{\text{S-DR}}(q_1, p_1, \theta), \tag{6.16}$$

$$p_1(q_1) = v - (v-\delta)F(q_1). \tag{6.17}$$

We prove the result in three steps. *Step 1:* Optimal q_1 in (6.16) is continuous and non-decreasing in p_1 as shown in Lemma 6.2.1 above. *Step 2:* From (6.17), we see that p_1 decreases in q_1. Also, $p_1 = v$ when $q_1 = 0$; $p_1 = \delta$ when $q_1 = \infty$; and $\delta \leq p_1 \leq v$. *Step 3:* From *Steps 1-2*, there must be a crossing point of (6.16) and (6.17). In addition, this crossing point is unique due to the non-decreasing property of $q_1(p_1)$ in (6.16) and the decreasing property of $p_1(q_1)$ in (6.17). So far, we have proved i).

With $\theta < \theta_{\max}$, i.e., $c + \alpha[v(1+\theta) - c] < v$, from the proof above, $p_1^{\text{S-DR}}(\theta) > c + \alpha[v(1+\theta) - c]$ and $q_1^{\text{S-DR}}(\theta) > 0$ are unique, and they must satisfy the two equations

$$p_1 = v - (v-\delta)F(q_1), \tag{6.18}$$

$$F(q_1) = \frac{p_1 - c - \alpha[v(1+\theta) - c]}{p_1 - \alpha[v(1+\theta) - c] - \alpha\delta}. \tag{6.19}$$

This proves claim ii.a).

Next, we prove ii.b) by contradiction. From (6.18), as θ changes, $q_1^{\text{S-DR}}(\theta)$ and $p_1^{\text{S-DR}}(\theta)$ must change in the opposite direction. Suppose that as θ increases, $q_1^{\text{S-DR}}(\theta)$ increases and $p_1^{\text{S-DR}}(\theta)$ decreases. Then, the right-hand side of (6.19) decreases as θ increases and $p_1^{\text{S-DR}}(\theta)$ decreases. However, the left-hand side of (6.19) increases as $q_1^{\text{S-DR}}(\theta)$ increases. This leads to a contradiction. Thus, $q_1^{\text{S-DR}}(\theta)$ must decrease while $p_1^{\text{S-DR}}(\theta)$ increases in θ.

With $\theta \geq \theta_{\max}$, i.e., $c + \alpha[v(1+\theta) - c] \geq v$, we know that $q_1^o(p_1)$ becomes positive only when $p_1 > c + \alpha[v(1+\theta) - c] \geq v$. So, the crossing point of (6.16) and (6.17) is given by $q_1^{\text{S-DR}}(\theta) = 0$ and $p_1^{\text{S-DR}}(\theta) = v$. This proves iii). ■

6.3 Proofs related to Chapter 4

Proof of Proposition 4.4.1

Proposition 4.4.1 follows from the discussion above it. ■

Proof of Lemma 4.4.2

Lemma 4.4.2 can be obtained immediately from the profit expressions listed in the proof of Proposition 4.4.3. ■

Proof of Proposition 4.4.3

To show Proposition 4.4.3, we consider two cases: $\delta_o/\delta_r \geq 2$ and $2 - \tau/c \leq \delta_o/\delta_r \leq 2$. In both cases, we evaluate the expected profits of both strategies for all nine possible stocking levels.

Case 1**:** $\delta_o/\delta_r \geq 2$. According to different stocking levels in the two channels, we have the following nine sub-cases to consider. We use π^{ROi} and π^{ORi} to denote the firm's expected profit under RO and OR strategies for case 1.i, respectively. Using equations (4.5) and (4.6), we can obtain the firm's expected profit for all sub-cases.

Case 1.1: $Q_r = \mu_r - \delta_r$, $Q_o = \mu_o - \delta_o$.

$$\pi^{\mathrm{RO1}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - (p-c)\delta_o,$$
$$\pi^{\mathrm{OR1}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - (p-c)\delta_o.$$

Case 1.2: $Q_r = \mu_r - \delta_r$, $Q_o = \mu_o$.

$$\pi^{\mathrm{RO2}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - pk\delta_o,$$
$$\pi^{\mathrm{OR2}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - pk\delta_o + (p-\tau)k\delta_r.$$

Case 1.3: $Q_r = \mu_r - \delta_r$, $Q_o = \mu_o + \delta_o$.

$$\pi^{\mathrm{RO3}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - c\delta_o,$$
$$\pi^{\mathrm{OR3}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - c\delta_o + (p-\tau)(1-k)\delta_r.$$

Case 1.4: $Q_r = \mu_r$, $Q_o = \mu_o - \delta_o$.

$$\pi^{\mathrm{RO4}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - (p-c)\delta_o + (p-\tau)k(1-k)\delta_r,$$
$$\pi^{\mathrm{OR4}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - (p-c)\delta_o.$$

Case 1.5: $Q_r = \mu_r$, $Q_o = \mu_o$.

$$\pi^{\mathrm{RO5}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - pk\delta_o + (p-\tau)k^2\delta_r,$$
$$\pi^{\mathrm{OR5}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - pk\delta_o + (p-\tau)k^2\delta_r.$$

Case 1.6: $Q_r = \mu_r$, $Q_o = \mu_o + \delta_o$.

$$\pi^{\mathrm{RO6}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - c\delta_o,$$
$$\pi^{\mathrm{OR6}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - c\delta_o + (p-\tau)k(1-k)\delta_r.$$

Case 1.7: $Q_r = \mu_r + \delta_r$, $Q_o = \mu_o - \delta_o$.

$$\pi^{\mathrm{RO7}} = (p-c)(\mu_r+\mu_o) - c\delta_r - (p-c)\delta_o + (p-\tau)(1-k)\delta_r,$$
$$\pi^{\mathrm{OR7}} = (p-c)(\mu_r+\mu_o) - c\delta_r - (p-c)\delta_o.$$

Case 1.8: $Q_r = \mu_r + \delta_r$, $Q_o = \mu_o$.

$$\pi^{\mathrm{RO8}} = (p-c)(\mu_r+\mu_o) - c\delta_r - pk\delta_o + (p-\tau)k\delta_r,$$
$$\pi^{\mathrm{OR8}} = (p-c)(\mu_r+\mu_o) - c\delta_r - pk\delta_o.$$

Case 1.9: $Q_r = \mu_r + \delta_r$, $Q_o = \mu_o + \delta_o$.

$$\pi^{\mathrm{RO9}} = (p-c)(\mu_r+\mu_o) - c\delta_r - c\delta_o,$$
$$\pi^{\mathrm{OR9}} = (p-c)(\mu_r+\mu_o) - c\delta_r - c\delta_o.$$

It is easy to see that $\pi^{\mathrm{RO1}} = \pi^{\mathrm{OR1}}$, $\pi^{\mathrm{RO2}} \leq \pi^{\mathrm{OR2}}$, $\pi^{\mathrm{RO3}} \leq \pi^{\mathrm{OR3}}$, $\pi^{\mathrm{RO4}} \geq \pi^{\mathrm{OR4}}$, $\pi^{\mathrm{RO5}} = \pi^{\mathrm{OR5}}$, $\pi^{\mathrm{RO6}} \leq \pi^{\mathrm{OR6}}$, $\pi^{\mathrm{RO7}} \geq \pi^{\mathrm{OR7}}$, $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR8}}$, and $\pi^{\mathrm{RO9}} = \pi^{\mathrm{OR9}}$. Next, we rank the above profit expressions based on the profit contribution margin of the product.

Suppose $p = 2c$. Then $\pi^{\mathrm{RO1}} = \pi^{\mathrm{OR1}}$, $\pi^{\mathrm{RO2}} = \pi^{\mathrm{OR8}}$, $\pi^{\mathrm{RO3}} = \pi^{\mathrm{OR7}}$, $\pi^{\mathrm{RO4}} = \pi^{\mathrm{OR6}}$, $\pi^{\mathrm{RO5}} = \pi^{\mathrm{OR5}}$, $\pi^{\mathrm{RO6}} = \pi^{\mathrm{OR4}}$, $\pi^{\mathrm{RO7}} = \pi^{\mathrm{OR3}}$, $\pi^{\mathrm{RO8}} = \pi^{\mathrm{OR2}}$ and $\pi^{\mathrm{RO9}} = \pi^{\mathrm{OR9}}$. Because $\pi^{\mathrm{RO}} = \max\{\pi^{\mathrm{RO1}}, \pi^{\mathrm{RO2}}, \pi^{\mathrm{RO3}}, \pi^{\mathrm{RO4}}, \pi^{\mathrm{RO5}}, \pi^{\mathrm{RO6}}, \pi^{\mathrm{RO7}}, \pi^{\mathrm{RO8}}, \pi^{\mathrm{RO9}}\}$ and $\pi^{\mathrm{OR}} = \max\{\pi^{\mathrm{OR1}}, \pi^{\mathrm{OR2}}, \pi^{\mathrm{OR3}}, \pi^{\mathrm{OR4}}, \pi^{\mathrm{OR5}}, \pi^{\mathrm{OR6}}, \pi^{\mathrm{OR7}}, \pi^{\mathrm{OR8}}, \pi^{\mathrm{OR9}}\}$, we have $\pi^{\mathrm{RO}} = \pi^{\mathrm{OR}}$.

Suppose $p < 2c$. We shall prove claim ii) by considering two cases: $p \leq c/(1-k)$ and $c/(1-k) < p \leq 2c$.

First, consider the case $p \leq c/(1-k)$. Using the condition $p - c \leq pk < c$, we see that $\pi^{\mathrm{RO1}} \geq \pi^{\mathrm{RO2}} \geq \pi^{\mathrm{RO3}}$, $\pi^{\mathrm{RO1}} \geq \pi^{\mathrm{RO6}} \geq \pi^{\mathrm{RO9}}$, $\pi^{\mathrm{RO4}} \geq \pi^{\mathrm{RO5}}$ and $\pi^{\mathrm{RO7}} \geq \pi^{\mathrm{RO8}}$. It follows that

$$\pi^{\mathrm{RO}} = \max\{\pi^{\mathrm{RO1}}, \pi^{\mathrm{RO4}}, \pi^{\mathrm{RO7}}\}.$$

Moreover, we can show that $\pi^{\mathrm{OR1}} \geq \pi^{\mathrm{OR4}} \geq \pi^{\mathrm{OR7}} \geq \pi^{\mathrm{OR9}}$, $\pi^{\mathrm{OR2}} \geq \pi^{\mathrm{OR5}}$, $\pi^{\mathrm{OR2}} \geq \pi^{\mathrm{OR3}} \geq \pi^{\mathrm{OR6}}$, and $\pi^{\mathrm{OR7}} \geq \pi^{\mathrm{OR8}}$. Thus,

$$\pi^{\mathrm{OR}} = \max\{\pi^{\mathrm{OR1}}, \pi^{\mathrm{OR2}}\}.$$

Because $\pi^{\mathrm{RO1}} = \pi^{\mathrm{OR1}}$, in order to rank π^{RO} and π^{OR}, it is sufficient to compare π^{OR2} vs. π^{RO4}, and π^{OR2} vs. π^{RO7}.

From the expressions of π^{OR2} and π^{RO7},

$$\pi^{\mathrm{OR2}} - \pi^{\mathrm{RO7}} = (p-c)(\delta_o - 2\delta_r) + \tau\delta_r - [p\delta_o - 2(p-\tau)\delta_r]k.$$

Therefore, when $k < k_1 = [(p-c)(\delta_o - 2\delta_r) + \tau\delta_r]/[p\delta_o - 2(p-\tau)\delta_r]$, we have $\pi^{\mathrm{OR2}} > \pi^{\mathrm{RO7}}$.

Similarly, from the expressions of π^{OR2} and π^{RO4},

$$\pi^{\mathrm{OR2}} - \pi^{\mathrm{RO4}} = (p-c)(\delta_o - \delta_r) + [p\delta_r k - p\delta_o k + (p-\tau)\delta_r k^2].$$

It is straightforward to verify that $p\delta_r k - p\delta_o k + (p-\tau)\delta_r k^2$ decreases in k for $0 \leq k \leq 1/2$. It must be true that, when $k \leq k_2$, $\pi^{\mathrm{OR2}} \geq \pi^{\mathrm{RO4}}$; otherwise $\pi^{\mathrm{OR2}} \leq \pi^{\mathrm{RO4}}$. Therefore, when $k \leq \min\{k_1, k_2\}$, $\pi^{\mathrm{OR}} \geq \pi^{\mathrm{RO}}$; otherwise $\pi^{\mathrm{OR}} \leq \pi^{\mathrm{RO}}$.

Next, consider the case $c/(1-k) < p \leq 2c$. Using the condition $c/(1-k) < p \leq 2c$, we can show that $\pi^{\mathrm{RO2}} \geq \pi^{\mathrm{RO1}}$, $\pi^{\mathrm{RO2}} \geq \pi^{\mathrm{RO3}}$ and $\pi^{\mathrm{RO4}} \geq \pi^{\mathrm{RO6}} \geq \pi^{\mathrm{RO9}}$. Therefore,

$$\pi^{\mathrm{RO}} = \max\{\pi^{\mathrm{RO2}}, \pi^{\mathrm{RO4}}, \pi^{\mathrm{RO5}}, \pi^{\mathrm{RO7}}, \pi^{\mathrm{RO8}}\}.$$

Similarly, we also can show that $\pi^{\mathrm{OR8}} \geq \pi^{\mathrm{OR9}}$, $\pi^{\mathrm{OR8}} \geq \pi^{\mathrm{OR7}}$, $\pi^{\mathrm{OR4}} \geq \pi^{\mathrm{OR1}}$, and $\pi^{\mathrm{OR2}} \geq \pi^{\mathrm{OR3}}$. Consequently,

$$\pi^{\mathrm{OR}} = \max\{\pi^{\mathrm{OR2}}, \pi^{\mathrm{OR4}}, \pi^{\mathrm{OR5}}, \pi^{\mathrm{OR6}}, \pi^{\mathrm{OR8}}\}.$$

Note that $\pi^{\mathrm{RO2}} \le \pi^{\mathrm{OR2}}$, $\pi^{\mathrm{RO4}} \ge \pi^{\mathrm{OR4}}$, $\pi^{\mathrm{RO5}} = \pi^{\mathrm{OR5}}$ and $\pi^{\mathrm{RO8}} \ge \pi^{\mathrm{OR8}}$. In order to obtain the desired result, it is sufficient to show either $\pi^{\mathrm{OR2}} \ge \pi^{\mathrm{RO4}}$, $\pi^{\mathrm{OR2}} \ge \pi^{\mathrm{RO8}}$, and $\pi^{\mathrm{OR2}} \ge \pi^{\mathrm{RO7}}$ or $\pi^{\mathrm{OR6}} \ge \pi^{\mathrm{RO4}}$, $\pi^{\mathrm{OR6}} \ge \pi^{\mathrm{RO8}}$, and $\pi^{\mathrm{OR6}} \ge \pi^{\mathrm{RO7}}$.

From the expressions of π^{OR2}, π^{RO4}, π^{RO7}, and π^{RO8} , we have

$$\begin{aligned}
\pi^{\mathrm{OR2}} - \pi^{\mathrm{RO4}} &= (p-c-pk)(\delta_o - \delta_r) + (p-\tau)\delta k^2 > 0,\\
\pi^{\mathrm{OR2}} - \pi^{\mathrm{RO7}} &= (2c-p)\delta_r + (p-c-pk)\delta_0 + (p-\tau)(2k-1)\delta_r\\
&\ge (2c-p)\delta_r + 2(p-c-pk)\delta_r + (p-\tau)(2k-1)\delta_r \ge 0,\\
\pi^{\mathrm{OR2}} - \pi^{\mathrm{RO8}} &= (2c-p)\delta_r \ge 0.
\end{aligned}$$

Therefore, $\pi^{\mathrm{OR}} \ge \pi^{\mathrm{RO}}$ for $k \in [0, 1/2]$ when $c/(1-k) < p \le 2c$.

Combining the two cases $p \le c/(1-k)$ and $c/(1-k) < p \le 2c$, we prove claim ii) for the case $\delta_o/\delta_r \ge 2$.

Suppose $p > 2c$. We shall prove claim iii) by consider two cases: $2c < p < c/k$ and $p \ge c/k$.

Suppose $2c < p < c/k$. In this case, $pk < p - c$ and $pk < c$. Using these conditions, we can show that $\pi^{\mathrm{RO5}} \ge \pi^{\mathrm{RO2}} \ge \pi^{\mathrm{RO1}}$, $\pi^{\mathrm{RO2}} \ge \pi^{\mathrm{RO3}}$,$\pi^{\mathrm{RO5}} \ge \pi^{\mathrm{RO6}} \ge \pi^{\mathrm{RO9}}$, $\pi^{\mathrm{RO5}} \ge \pi^{\mathrm{RO4}}$, and $\pi^{\mathrm{RO8}} \ge \pi^{\mathrm{RO7}}$. Therefore,

$$\pi^{\mathrm{RO}} = \max\{\pi^{\mathrm{RO5}}, \pi^{\mathrm{RO8}}\}.$$

It is also straightforward to show that $\pi^{\mathrm{OR8}} \ge \pi^{\mathrm{OR9}}$, $\pi^{\mathrm{OR8}} \ge \pi^{\mathrm{OR7}}$, $\pi^{\mathrm{OR8}} \ge \pi^{\mathrm{OR1}}$ and $\pi^{\mathrm{OR8}} \ge \pi^{\mathrm{OR4}}$. Hence,

$$\pi^{\mathrm{OR}} = \max\{\pi^{\mathrm{OR2}}, \pi^{\mathrm{OR3}}, \pi^{\mathrm{OR5}}, \pi^{\mathrm{OR6}}, \pi^{\mathrm{OR8}}\}.$$

Notice that

$$\begin{aligned}
\pi^{\mathrm{RO8}} - \pi^{\mathrm{OR2}} &= (p-2c)\delta_r > 0,\\
\pi^{\mathrm{RO8}} - \pi^{\mathrm{OR3}} &= (p-2c)\delta_r + (c-pk)\delta_o + (p-\tau)(2k-1)\delta_r\\
&> (p-2c)\delta_r + 2(c-pk)\delta_r + (p-\tau)(2k-1)\delta_r \geq 0,\\
\pi^{\mathrm{RO8}} - \pi^{\mathrm{OR6}} &= (pk-c)(\delta_r-\delta_o) + (p-\tau)\delta_r k^2 > 0.
\end{aligned}$$

Together with the facts that $\pi^{\mathrm{RO5}} = \pi^{\mathrm{OR5}}$ and $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR8}}$, we complete the proof for claim iii) when $2c < p < c/k$.

Suppose $p \geq c/k$. It then follows that $\pi^{\mathrm{RO9}} \geq \pi^{\mathrm{RO6}} \geq \pi^{\mathrm{RO3}} \geq \pi^{\mathrm{RO2}} \geq \pi^{\mathrm{RO1}}$, $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{RO5}}$, and $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{RO7}} \geq \pi^{\mathrm{RO4}}$. Consequently,

$$\pi^{\mathrm{RO}} = \max\{\pi^{\mathrm{RO8}}, \pi^{\mathrm{RO9}}\}.$$

We also can show that when $p \geq c/k$, $\pi^{\mathrm{OR9}} \geq \pi^{\mathrm{OR8}} \geq \pi^{\mathrm{OR7}}$, $\pi^{\mathrm{OR9}} \geq \pi^{\mathrm{OR4}}$, $\pi^{\mathrm{OR9}} \geq \pi^{\mathrm{OR1}}$, $\pi^{\mathrm{OR3}} \geq \pi^{\mathrm{OR2}}$, and $\pi^{\mathrm{OR6}} \geq \pi^{\mathrm{OR5}}$. Then, $\pi^{\mathrm{OR}} = \max\{\pi^{\mathrm{OR3}}, \pi^{\mathrm{OR6}}, \pi^{\mathrm{OR9}}\}$. Since $\pi^{\mathrm{RO9}} = \pi^{\mathrm{OR9}}$, we just need to compare π^{RO8} vs. π^{OR3}, and π^{RO8} vs. π^{OR6}.

Notice that

$$\pi^{\mathrm{RO8}} - \pi^{\mathrm{OR3}} = c(\delta_o - 2\delta_r) + \tau\delta_r - [p\delta_o - 2(p-\tau)\delta_r]k.$$

It is straightforward to see that when $k < k_3 = [c(\delta_o - 2\delta_r) + \tau\delta_r]/[p\delta_o - 2(p-\tau)\delta_r]$, we have $\pi^{\mathrm{RO8}} > \pi^{\mathrm{OR3}}$. We also know that

$$\pi^{\mathrm{RO8}} - \pi^{\mathrm{OR6}} = c(\delta_o - \delta_r) + pk(\delta_r - \delta_o) + (p-\tau)k^2\delta_r.$$

It is easy to verify that $pk(\delta_r - \delta_o) + (p-\tau)k^2\delta_r$ decreases in k for $k \in [0, 1/2]$. Since $\delta_o \geq 2\delta_r$, we know $\pi^{\mathrm{RO8}} - \pi^{\mathrm{OR6}} > 0$ when $k = 0$. Thus, when $k \leq k_4$, we have $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR6}}$. Therefore, when $k \leq \min\{k_3, k_4\}$, $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$; otherwise $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$. Combining the two cases $2c < p < c/k$ and $p \geq c/k$, by now we already prove claim iii) for case 1 ($\delta_o/\delta_r \geq 2$).

CASE 2: $2 - \tau/c \leq \delta_o/\delta_r \leq 2$. Similar to case 1, we have nine sub-cases to consider. Again, using equations (4.5) and (4.6), we can obtain the firm's expected profit for all sub-cases.

Case 2.1: $Q_r = \mu_r - \delta_r$, $Q_o = \mu_o - \delta_o$.

$$\pi^{\text{RO1}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - (p-c)\delta_o,$$
$$\pi^{\text{OR1}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - (p-c)\delta_o.$$

Case 2.2: $Q_r = \mu_r - \delta_r$, $Q_o = \mu_o$.

$$\pi^{\text{RO2}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - pk\delta_o,$$
$$\pi^{\text{OR2}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - pk\delta_o + (p-\tau)[\delta_r k(1-2k) + \delta_o k^2].$$

Case 2.3: $Q_r = \mu_r - \delta_r$, $Q_o = \mu_o + \delta_o$.

$$\pi^{\text{RO3}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - c\delta_o,$$
$$\pi^{\text{OR3}} = (p-c)(\mu_r + \mu_o) - (p-c)\delta_r - c\delta_o + (p-\tau)[\delta_r(4k^2 - 3k + 1) + \delta_o(1-2k)k].$$

Case 2.4: $Q_r = \mu_r$, $Q_o = \mu_o - \delta_o$.

$$\pi^{\text{RO4}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - (p-c)\delta_o + (p-\tau)k(1-k)\delta_r,$$
$$\pi^{\text{OR4}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - (p-c)\delta_o.$$

Case 2.5: $Q_r = \mu_r$, $Q_o = \mu_o$.

$$\pi^{\text{RO5}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - pk\delta_o + (p-\tau)k^2\delta_r,$$
$$\pi^{\text{OR5}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - pk\delta_o + (p-\tau)k^2\delta_r.$$

Case 2.6: $Q_r = \mu_r$, $Q_o = \mu_o + \delta_o$.

$$\pi^{\text{RO6}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - c\delta_o,$$
$$\pi^{\text{OR6}} = (p-c)(\mu_r + \mu_o) - pk\delta_r - c\delta_o + (p-\tau)k(1-k)\delta_r.$$

Case 2.7: $Q_r = \mu_r + \delta_r$, $Q_o = \mu_o - \delta_o$.

$$\pi^{\text{RO7}} = (p-c)(\mu_r + \mu_o) - c\delta_r - (p-c)\delta_o + (p-\tau)[\delta_r(4k^2 - 3k + 1) + \delta_o(1-2k)k],$$
$$\pi^{\text{OR7}} = (p-c)(\mu_r + \mu_o) - c\delta_r - (p-c)\delta_o.$$

Case 2.8: $Q_r = \mu_r + \delta_r$, $Q_o = \mu_o$.

$$\pi^{\text{RO8}} = (p-c)(\mu_r + \mu_o) - c\delta_r - pk\delta_o + (p-\tau)[\delta_r(1-2k)k + \delta_o k^2],$$
$$\pi^{\text{OR8}} = (p-c)(\mu_r + \mu_o) - c\delta_r - pk\delta_o.$$

Case 2.9: $Q_r = \mu_r + \delta_r$, $Q_o = \mu_o + \delta_o$.

$$\pi^{\text{RO9}} = (p-c)(\mu_r + \mu_o) - c\delta_r - c\delta_o,$$
$$\pi^{\text{OR9}} = (p-c)(\mu_r + \mu_o) - c\delta_r - c\delta_o.$$

Note that, compared with the expressions in case 1, only π^{RO7}, π^{RO8}, π^{OR2}, and π^{OR3} are different.

It can be verified that $\pi^{\text{RO1}} = \pi^{\text{OR1}}$, $\pi^{\text{RO2}} \leq \pi^{\text{OR2}}$, $\pi^{\text{RO3}} \leq \pi^{\text{OR3}}$, $\pi^{\text{RO4}} \geq \pi^{\text{OR4}}$, $\pi^{\text{RO5}} = \pi^{\text{OR5}}$, $\pi^{\text{RO6}} \leq \pi^{\text{OR6}}$,$\pi^{\text{RO7}} \geq \pi^{\text{OR7}}$, $\pi^{\text{RO8}} \geq \pi^{\text{OR8}}$, and $\pi^{\text{RO9}} = \pi^{\text{OR9}}$. As in case 1, we shall rank the above expected profit expressions based on the profit contribution margin of the product.

Suppose $p = 2c$. Notice that the relations $\pi^{\text{RO1}} = \pi^{\text{OR1}}$, $\pi^{\text{RO2}} = \pi^{\text{OR8}}$, $\pi^{\text{RO3}} = \pi^{\text{OR7}}$, $\pi^{\text{RO4}} = \pi^{\text{OR6}}$, $\pi^{\text{RO5}} = \pi^{\text{OR5}}$, $\pi^{\text{RO6}} = \pi^{\text{OR4}}$, $\pi^{\text{RO7}} = \pi^{\text{OR3}}$, $\pi^{\text{RO8}} = \pi^{\text{OR2}}$ and $\pi^{\text{RO9}} = \pi^{\text{OR9}}$ continue to hold. Thus, $\pi^{\text{RO}} = \pi^{\text{OR}}$.

Suppose $p < 2c$. Again, We shall prove claim ii by considering two cases: $p \leq c/(1-k)$ and $c/(1-k) < p \leq 2c$.

First, consider the case $p \leq c/(1-k)$. It can be verified that $\pi^{\text{RO1}} \geq \pi^{\text{RO2}} \geq \pi^{\text{RO3}} \geq \pi^{\text{RO6}} \geq \pi^{\text{RO9}}$, $\pi^{\text{RO4}} \geq \pi^{\text{RO5}}$, and $\pi^{\text{RO7}} \geq \pi^{\text{RO8}}$. Hence,

$$\pi^{\text{RO}} = \max\{\pi^{\text{RO1}}, \pi^{\text{RO4}}, \pi^{\text{RO7}}\}.$$

Using the conditions $p \leq c/(1-k)$ and $\delta_r \leq \delta_o < 2\delta_r$, we can show that $\pi^{\text{OR1}} \geq \pi^{\text{OR4}} \geq \pi^{\text{OR7}} \geq \pi^{\text{OR9}}$, $\pi^{\text{OR3}} \geq \pi^{\text{OR6}}$, $\pi^{\text{OR2}} \geq \pi^{\text{OR5}}$, and $\pi^{\text{OR7}} \geq \pi^{\text{OR8}}$. Thus,

$$\pi^{\text{OR}} = \max\{\pi^{\text{OR1}}, \pi^{\text{OR2}}, \pi^{\text{OR3}}\}.$$

It is straightforward to see that $\pi^{\text{RO7}} > \pi^{\text{OR3}}$. Together with the fact $\pi^{\text{RO1}} = \pi^{\text{OR1}}$, we just need to compare π^{OR2} vs. π^{OR4}, and π^{OR2} vs. π^{OR7} in order to show claim ii. Note that

$$\pi^{\text{OR2}}-\pi^{\text{RO7}} = (2c-p)\delta_r+(p-c-pk)\delta_o+(p-\tau)[\delta_r(1-2k)(2k-1)-2\delta_r k^2+\delta_o k^2-k(1-2k)\delta_o].$$

Clearly, $(p-\tau)[\delta_r(1-2k)(2k-1) - 2\delta_r k^2 + \delta_o k^2 - k(1-2k)\delta_o] - pk\delta_o$, as a quadratic function of k, first increases in k when $k \leq [2(p-\tau)(2\delta_r - \delta_o) - p\delta_o]/[6(p-\tau)(2\delta_r - \delta_o)]$, and then decreases in k for $k \in [0, 1/2]$. Notice that $\pi^{\text{OR2}} - \pi^{\text{RO7}} > 0$ when $k = 0$. It follows that, when $k \leq k_5$, $\pi^{\text{OR2}} \geq \pi^{\text{RO7}}$; otherwise $\pi^{\text{OR2}} \leq \pi^{\text{RO7}}$.

Also notice that

$$\pi^{\text{OR2}} - \pi^{\text{RO4}} = (\delta_o - \delta_r)[p - c - pk + (p-\tau)k^2].$$

It is easy to show that $p - c - pk + (p-\tau)k^2$ decreases in k for $k \in [0, 1/2]$. Therefore, when $k \leq \min\{k_5, k_6\}$, $\pi^{\text{OR2}} \geq \pi^{\text{RO4}}, \pi^{\text{OR2}} \geq \pi^{\text{RO7}}$. Combining with the facts that $\pi^{\text{RO1}} = \pi^{\text{OR1}}$ and $\pi^{\text{RO7}} \geq \pi^{\text{OR3}}$, we know that when $k \leq \min\{k_5, k_6\}$, $\pi^{\text{OR}} \geq \pi^{\text{RO}}$; otherwise $\pi^{\text{OR}} \leq \pi^{\text{RO}}$.

Suppose $c/(1-k) < p \leq 2c$. Note that when $c/(1-k) < p \leq 2c$, $\pi^{\text{RO2}} \geq \pi^{\text{RO1}}$, $\pi^{\text{RO2}} \geq \pi^{\text{RO3}}$, $\pi^{\text{RO4}} \geq \pi^{\text{RO6}} \geq \pi^{\text{RO9}}$, $\pi^{\text{OR8}} \geq \pi^{\text{OR9}}$, $\pi^{\text{OR5}} \geq \pi^{\text{OR6}}$, $\pi^{\text{OR8}} \geq \pi^{\text{OR7}}$, and $\pi^{\text{OR4}} \geq \pi^{\text{OR1}}$. Then,

$$\pi^{\text{RO}} = \max\{\pi^{\text{RO2}}, \pi^{\text{RO4}}, \pi^{\text{RO5}}, \pi^{\text{RO7}}, \pi^{\text{RO8}}\},$$
$$\pi^{\text{OR}} = \max\{\pi^{\text{OR2}}, \pi^{\text{OR3}}, \pi^{\text{OR4}}, \pi^{\text{OR5}}, \pi^{\text{OR8}}\}.$$

Note that

$$\pi^{\text{OR2}} - \pi^{\text{RO4}} = (\delta_o - \delta_r)[p - c - pk + (p-\tau)k^2] > 0,$$
$$\pi^{\text{OR2}} - \pi^{\text{RO8}} = (2c - p)\delta_r \geq 0,$$
$$\pi^{\text{RO7}} - \pi^{\text{OR3}} = (2c - p)(\delta_o - \delta_r) \geq 0.$$

Moreover, $\pi^{\text{OR2}} \geq \pi^{\text{RO2}}$, $\pi^{\text{RO4}} \geq \pi^{\text{OR4}}$, $\pi^{\text{RO8}} \geq \pi^{\text{OR8}}$, and $\pi^{\text{OR5}} = \pi^{\text{RO5}}$. Consequently, it is sufficient to compare π^{OR2} vs. π^{RO7} in order to rank π^{RO} and π^{OR}. From the proof of claim ii when $p \leq c/(1-k)$, we already know that, when $k \leq k_5$, $\pi^{\text{OR2}} \geq \pi^{\text{RO7}}$; otherwise $\pi^{\text{OR2}} \leq \pi^{\text{RO7}}$. Therefore, when $k \leq k_5$, $\pi^{\text{OR}} \geq \pi^{\text{RO}}$; otherwise $\pi^{\text{OR}} \leq \pi^{\text{RO}}$. Combining the two cases $p \leq c/(1-k)$ and $c/(1-k) < p \leq 2c$, we prove claim ii for case 2.

Suppose $p > 2c$. As before, we also prove claim iii) in two cases: $2c < p < c/k$ and $p \geq c/k$.

Suppose $2c < p < c/k$. It is straightforward to verify that when $2c < p < c/k$, we have $\pi^{\text{RO5}} \geq \pi^{\text{RO2}} \geq \pi^{\text{RO1}}$, $\pi^{\text{RO5}} \geq \pi^{\text{RO6}} \geq \pi^{\text{RO9}}$, $\pi^{\text{RO2}} \geq \pi^{\text{RO3}}$ and $\pi^{\text{RO2}} \geq \pi^{\text{RO3}}$. Thus,

$$\pi^{\text{RO}} = \{\pi^{\text{RO5}}, \pi^{\text{RO7}}, \pi^{\text{RO8}}\}.$$

It is also easy to show that $\pi^{\text{OR8}} \geq \pi^{\text{OR1}}$, $\pi^{\text{OR8}} \geq \pi^{\text{OR4}}$, $\pi^{\text{OR8}} \geq \pi^{\text{OR7}}$, and $\pi^{\text{OR8}} \geq \pi^{\text{OR9}}$. Thus,

$$\pi^{\text{OR}} = \max\{\pi^{\text{OR2}}, \pi^{\text{OR3}}, \pi^{\text{OR5}}, \pi^{\text{OR6}}, \pi^{\text{OR8}}\}.$$

Notice that

$$\pi^{\text{RO8}} - \pi^{\text{OR2}} = (2p - c)\delta_r > 0,$$
$$\pi^{\text{RO8}} - \pi^{\text{OR6}} = (c - pk)(\delta_o - \delta_r) + (p - \tau)[\delta_r k(1 - k) + \delta_o k^2] > 0,$$
$$\pi^{\text{RO7}} - \pi^{\text{OR3}} = (p - 2c)(\delta_r - \delta_o) < 0.$$

Together with the facts that $\pi^{\text{RO5}} \geq \pi^{\text{OR5}}$ and $\pi^{\text{RO8}} \geq \pi^{\text{OR8}}$, we just need to compare π^{RO8} vs. π^{OR3} in order to obtain the desired result. Also note that

$$\pi^{\text{RO8}} - \pi^{\text{OR3}} = (p-2c)\delta_r + (c-pk)\delta_o + (p-\tau)[\delta_r(1-2k)(2k-1) - 2\delta_r k^2 + \delta_o k^2 - k(1-2k)\delta_o].$$

It is easy to see that $\pi^{\text{RO8}} - \pi^{\text{OR3}}$ increases in k when $k \leq [2(p-\tau)(2\delta_r - \delta_o) - p\delta_o]/[6(p-\tau)(2\delta_r - \delta_o)]$, otherwise it decreases in k. However, when $k = 0$, $\pi^{\text{RO8}} - \pi^{\text{OR3}} \geq 0$. It follows that $\pi^{\text{RO8}} \geq \pi^{\text{OR3}}$ when $k \leq k_7$; otherwise $\pi^{\text{RO8}} \leq \pi^{\text{OR3}}$. Thus, we conclude that when $k \leq k_7$, $\pi^{\text{RO}} \geq \pi^{\text{OR}}$, otherwise $\pi^{\text{RO}} \leq \pi^{\text{OR}}$.

Suppose $p \geq c/k$. Note that when $p \geq c/k$, $\pi^{\text{RO9}} \geq \pi^{\text{RO6}} \geq \pi^{\text{RO3}} \geq \pi^{\text{RO2}} \geq \pi^{\text{RO1}}$, $\pi^{\text{RO7}} \geq \pi^{\text{RO4}}$, and $\pi^{\text{RO8}} \geq \pi^{\text{RO5}}$. Therefore,

$$\pi^{\text{RO}} = \max\{\pi^{\text{RO7}}, \pi^{\text{RO8}}, \pi^{\text{RO9}}\}.$$

Moreover, we can also show that $\pi^{\text{OR9}} \geq \pi^{\text{OR8}} \geq \pi^{\text{OR7}} \geq \pi^{\text{OR4}} \geq \pi^{\text{OR1}}$, $\pi^{\text{OR3}} \geq \pi^{\text{OR2}}$ and $\pi^{\text{OR6}} \geq \pi^{\text{OR5}}$. Thus,

$$\pi^{\text{OR}} = \max\{\pi^{\text{OR3}}, \pi^{\text{OR6}}, \pi^{\text{OR9}}\}.$$

Combining with the facts $\pi^{\text{RO7}} \leq \pi^{\text{OR3}}$ and $\pi^{\text{RO9}} = \pi^{\text{OR9}}$, we just need to compare π^{RO8} vs. π^{OR3} and π^{RO8} vs. π^{OR6} in order to obtain the desired result. Note that

$$\pi^{\text{RO8}} - \pi^{\text{OR6}} = (\delta_r - \delta_o)(pk - c - pk^2 + \tau k^2).$$

It is easy to verify that $\pi^{\text{RO8}} - \pi^{\text{OR6}}$ decreases in k for $k \in [0, 1/2]$ and $\pi^{\text{RO8}} - \pi^{\text{OR6}} > 0$

when $k = 0$. So, $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR6}}$ when $k \leq k_8$. From the proof of claim iii) for the case $2c < p < c/k$, we know that $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR3}}$ when $k \leq k_7$. Hence, when $k \leq \min\{k_7, k_8\}$, we have $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR6}}$ and $\pi^{\mathrm{RO8}} \geq \pi^{\mathrm{OR3}}$. Therefore, when $k \leq \min\{k_7, k_8\}$, $\pi^{\mathrm{RO}} \geq \pi^{\mathrm{OR}}$, otherwise $\pi^{\mathrm{RO}} \leq \pi^{\mathrm{OR}}$. This completes our proof. ■

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